



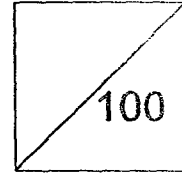
# 聖嬰中學

## HOLY INNOCENTS' HIGH SCHOOL

Name of Student

Class

Index Number



**END-OF-YEAR EXAMINATION 2015  
SECONDARY 3 EXPRESS  
ADDITIONAL MATHEMATICS**

4047

**Date: 6 Oct 2015**

**Duration: 2 hr 30 min**

*Additional Materials:* **7 sheets of writing papers  
1 string**

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number on all the work you hand in.  
Write in dark blue or black pen.  
You may use an HB pencil for any diagrams or graphs.  
Do not use paper clips, glue or correction tape/fluid.

Answer **ALL** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown clearly and neatly.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is **100**.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142.

**Set by:** Ms Lua Bee Hian

**Vetted by:** Mrs Rajammal Nathan and Mdm Hayati

Answer all questions.

1. (i) Sketch the graph of  $y = 2 + |3x + 1|$ . [3]

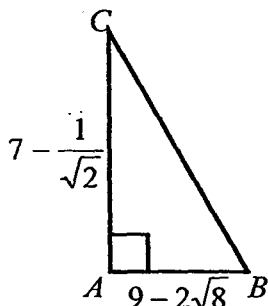
(ii) Find the set of values of the constant  $k$  for which the line  $y = kx + 4$  intersects the graph  $y = 2 + |3x + 1|$  at two distinct points. [2]

2. A curve has the equation  $y = 5x^2 - 4x + k$ , where  $k$  is a constant.

(i) In the case where  $k = -9$ , find the set of values of  $x$  for which  $y > 0$ . [3]

(ii) Find the value of  $k$  for which the line  $y = 6x + 4$  is a tangent to the curve. [3]

3.



The diagram shows a right-angled triangle  $ABC$  in which  $AB = (9 - 2\sqrt{8})$  cm and  $AC = \left(7 - \frac{1}{\sqrt{2}}\right)$  cm.

Express in the form of  $a + b\sqrt{2}$ , where  $a$  and  $b$  are rational numbers,

(i) the area of the triangle, [3]

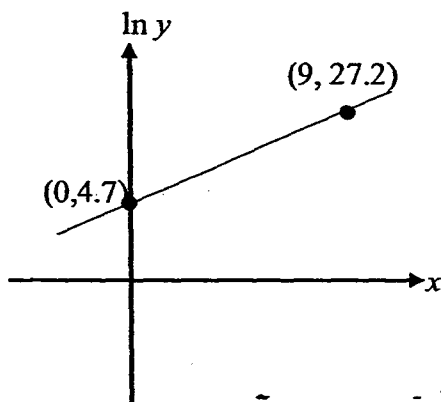
(ii)  $BC^2$ . [3]

4. Without using a calculator, find the value of

(i)  $15^x$ , given that  $75^{x-2} = 3^{2-x}$ , [4]

(ii)  $p$ , given that  $(\sqrt{5})^9 + (\sqrt{5})^7 + (\sqrt{5})^5 + (\sqrt{5})^3 - 155\sqrt{5} = 5^p$ . [3]

5.



The variables  $x$  and  $y$  are connected by the equation  $y = ab^x$ , where  $a$  and  $b$  are constants. Experimental values of  $x$  and  $y$  were obtained. The diagram above shows the straight line graph, passing through the points  $(0, 4.7)$  and  $(9, 27.2)$ , obtained by plotting  $\ln y$  against  $x$ .

Estimate

- (i) the value, to 2 significant figures, of  $a$  and of  $b$ , [6]
- (ii) the value of  $y$  when  $x = 2$ . [2]

6. The mass,  $m$  grams, of a radioactive substance, present at time  $t$  years after first being observed, is given by the formula  $m = 195(0.8)^t$ .

- (i) Find
- (a) the initial mass of the substance, [1]
- (b) the mass of the substance when  $t = 6$ , [1]
- (c) the value of  $t$  when the mass of the substance is  $\frac{1}{4}$  of its initial mass. [4]
- Give your answers correct to three significant figures.
- (ii) Explain why the mass of the substance can never be more than 195. [1]
- (iii) Sketch the graph of  $m$  against  $t$ . [1]

7. A circle,  $C_1$ , has equation  $x^2 + y^2 + 8x - 12y + 27 = 0$ .

(i) Find the radius and the coordinates of the centre of  $C_1$ . [3]

A second circle,  $C_2$  cuts  $C_1$  on the  $y$  axis at the points  $F$  and  $G$ , where  $FG$  is the diameter of  $C_2$ .

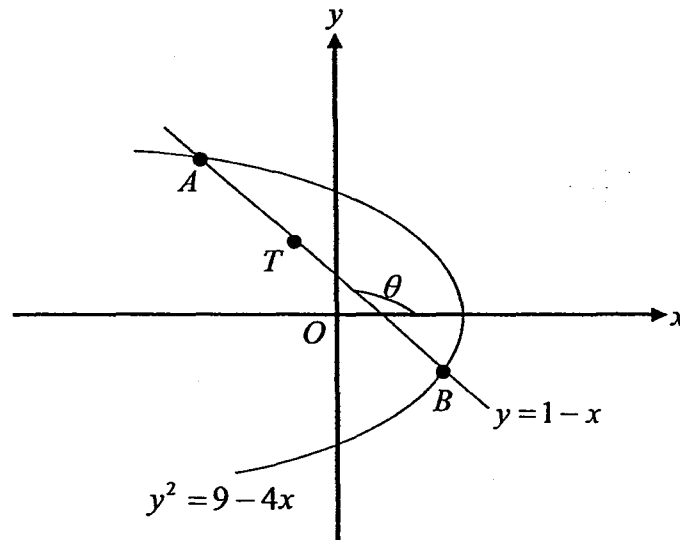
(ii) Find the equation of  $C_2$  in the form of  $(x-a)^2 + (y-b)^2 = r^2$ . [4]

(iii) Explain why the point  $(-3, 3)$  lies within only one of the circles  $C_1$  and  $C_2$ . [2]

8. (a) Solve  $2x^3 + 3x^2 - 11x - 6 = 0$ . [5]

(b) Express  $\frac{2x+3}{(x+1)(x^2-5)}$  in partial fractions. [5]

9. The diagram shows part of the curve  $y^2 = 9 - 4x$ , which meets the line  $y = 1 - x$  at the points  $A$  and  $B$ .



(i) Find

(a) the value of  $\theta$  in degrees, [2]

(b) the coordinates of  $A$  and  $B$ , [4]

(c) the area of triangle  $ABC$  where point  $C$  is  $(0, -3)$ . [1]

(ii) A perpendicular line passes through the point  $T$ , where  $T$  lies on the line  $y = 1 - x$  and the ratio of  $AT : TB$  is  $1 : 2$ . Find the equation of the perpendicular line. [3]

10. (a) Solve  $2\log_7 p = 3 + \log_p 49$ . [5]

(b) Given that  $\log_3 x = a$  and  $\log_9 y = b$ , express in terms of  $a$  and  $b$ ,

(i)  $\log_x 9y$ , [4]

(ii)  $x^3 y$ . [3]

11. (a) The roots of the quadratic equation  $3x^2 - 5x + 1 = 0$  are  $\alpha$  and  $\beta$ .

(i) Show that  $\frac{\alpha^2 + \beta^2}{\alpha\beta} = 6\frac{1}{3}$ . [4]

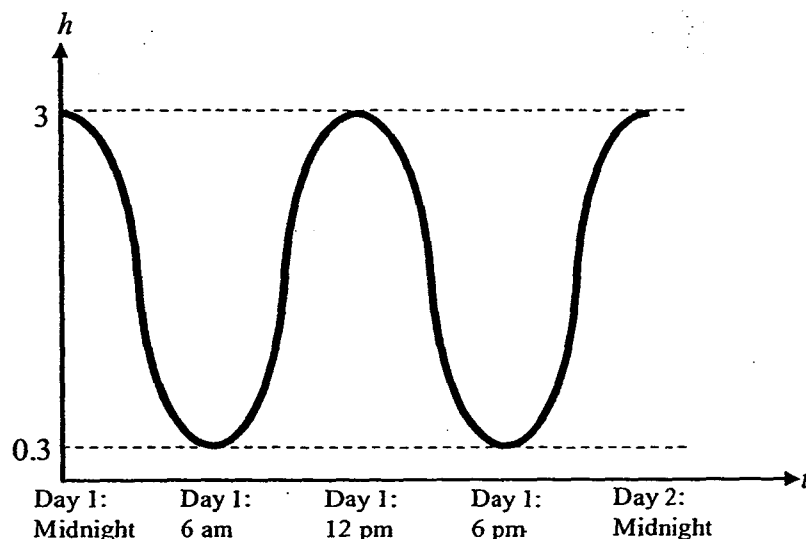
(ii) Find a quadratic equation whose roots are  $1 - \frac{\alpha}{\beta}$  and  $1 - \frac{\beta}{\alpha}$ . [4]

(b) (i) Without using a calculator, evaluate  $\tan 45^\circ(\sin 60^\circ + \cos 30^\circ)$ . [2]

(ii) Given that  $\sin A = \frac{3}{5}$  and that  $A$  is acute, find the value of  $3 \tan A + \cos\left(\frac{\pi}{2} - A\right)$ . [2]

12. (a) Solve  $5\operatorname{cosec}x + 11 = 0$  for  $0 \leq x \leq 2\pi$ . [4]

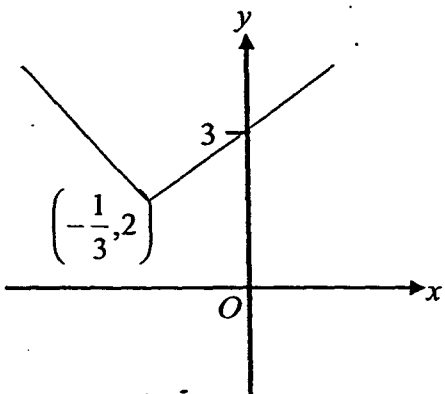
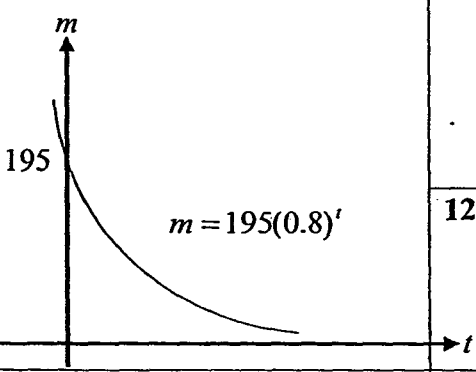
(b)

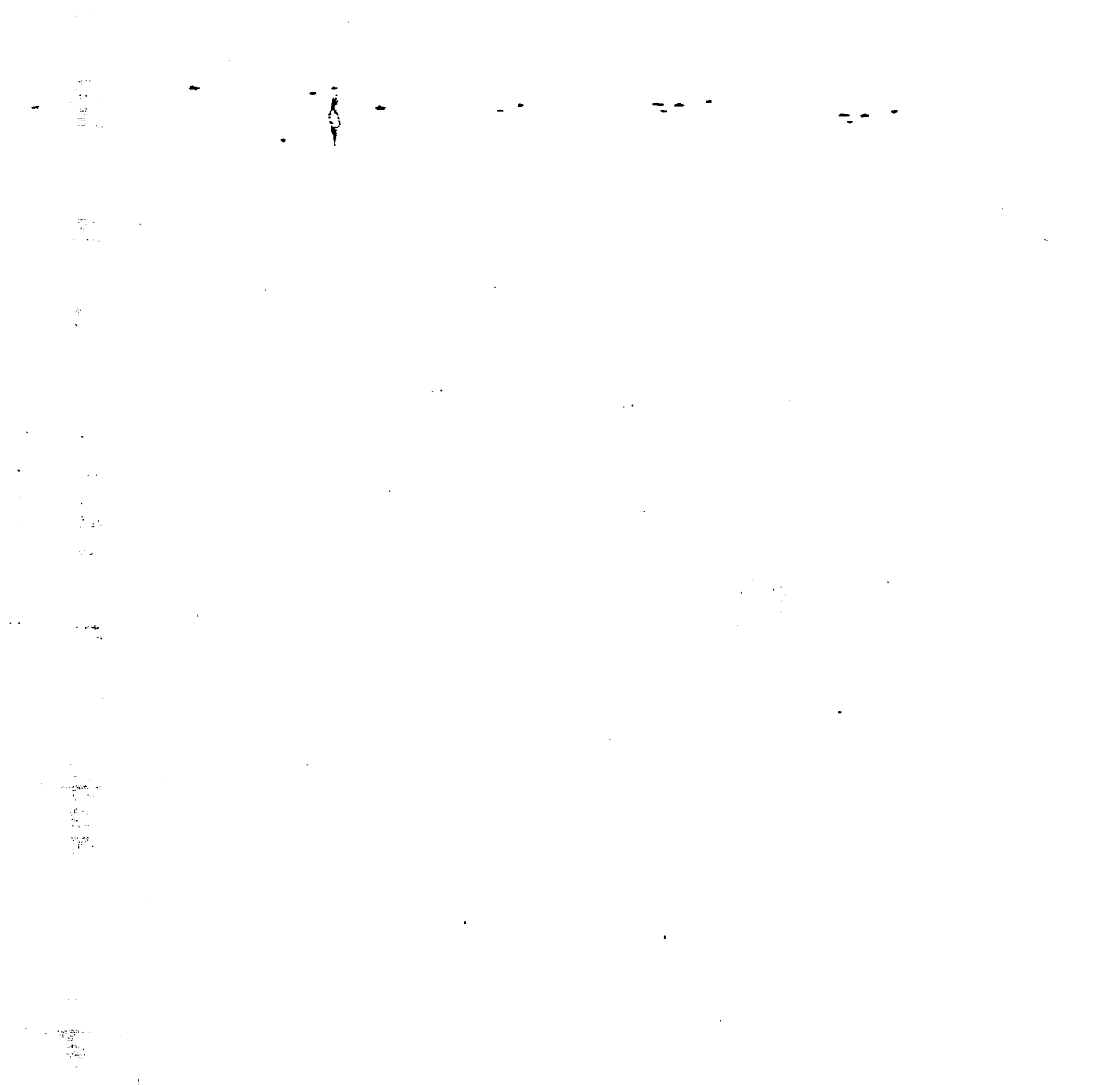


The diagram shows the graph obtained on a particular day when the heights,  $h$  metres of the tides are recorded at a beach on the east of Singapore over time,  $t$  hours. The height is modelled by the equation  $h = a + b \cos kt$ , where  $a$ ,  $b$  and  $k$  are constants.

(i) Show that the value of  $k$  is  $\frac{\pi}{6}$  radians per hour. [1]

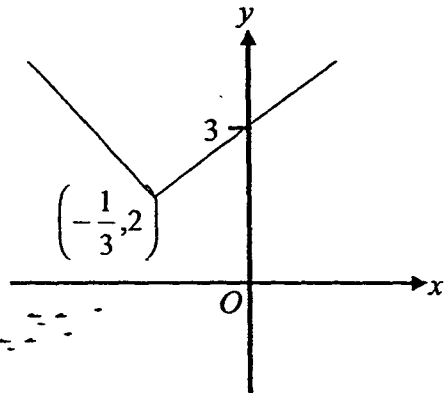
(ii) Find the value of  $a$  and of  $b$ . [2]

1	(i)		7	(ii) (iii)	Equation of $C_2$ is $x^2 + (y - 6)^2 = 3^2$ Circle 2: when $x = -3$ and $y = 3$ , $(-3)^2 + (3 - 6)^2 = 18$ , which is more than $3^2$ but less than $5^2$ .  Therefore the point $(-3, 3)$ lies in $C_1$ only.  <u>Alternate method</u> Circle 1: radius = 5 units Circle 2: radius = 3 units Distance from centre of Circle 1 to point $= \sqrt{(-4 + 3)^2 + (6 - 3)^2}$ $= \sqrt{10}$ units Since $3 < \sqrt{10} < 5$ , it lies within circle 1.
	(ii)	$-3 < k < 3$			
2	(i)	$x > 1\frac{4}{5}$ or $x < -1$			
	(ii)	$k = 9$			
3	(i)	$\frac{67}{2} - \frac{65}{4}\sqrt{2}$ cm <sup>2</sup>			
	(ii)	$BC^2 = \frac{325}{2} - 79\sqrt{2}$			
4	(i)	$15^x = 225$	8	(a)	$x = 2$ or $x = -\frac{1}{2}$ or $x = -3$
	(ii)	$p = 4.5$		(b)	$\frac{2x+3}{(x+1)(x^2-5)} = -\frac{1}{4(x+1)} + \frac{x+7}{4(x^2-5)}$
5	(i)	$a \approx 110$ and $b \approx 12$			
	(ii)	$y \approx 16300(3s.f.)$	9	(i)	(a) $\theta = 135^\circ$ (b) Coordinates of A is $(-4, 5)$ and coordinates of B is $(2, -1)$ . (c) 12 units <sup>2</sup> $y = x + 5$
6	(i)	(a) $m = 195$ gram (b) $m \approx 51.1$ gram (c) $t \approx 6.21$ years		(ii)	
	(ii)	Since the mass decreases with time, thus, the mass cannot never be more than 195 grams.  <i>Possible acceptable answer:</i> $t \rightarrow \infty, 0.8^t \rightarrow 0, 195(0.8^t) \rightarrow 0$ Therefore the mass can never be more than 195	10	(i)	$p \approx 0.378$ or $p = 49$
	(iii)		11	(ii)	(a) $\log_x 9y = \frac{2+2b}{a}$ (b) $3^{3a+2b}$
			11	(a)	(i) $\frac{\alpha^2 + \beta^2}{\alpha\beta} = 6\frac{1}{3}$ (ii) $x^2 + \frac{13}{3}x - \frac{13}{3} = 0$
				(b)	(i) $\sqrt{3}$ (ii) $2\frac{17}{20}$
			12	(a)	$x \approx 3.61$ or $x \approx 5.81$
				(b)	(i) Period = 12 hours $\rightarrow \cos(\frac{\pi}{6}t)$  Therefore $k = \frac{\pi}{6}$ (shown)
7	(i)	centre of circle = $(-4, 6)$ Radius = 5 units		(ii)	$a = 1.65, b = 1.35$







1	(i)		B1: General shape  B1: Labelled y-intercept or coordinates of y-axis  B1: Labelled - 'turning' pt or coordinates	General shape must be in 2 <sup>nd</sup> quad.  Line passing through y-intercept must be -ve gradient.
	(ii)	$-3 < k < 3$	B2	Show $-3 < k$ or $k < 3$ B1 each
2	(i)	$5x^2 - 4x - 9 > 0$ $(5x - 9)(x + 1) > 0$ $5x - 9 > 0 \quad \text{or} \quad x + 1 < 0$ $x > 1\frac{4}{5} \quad \quad \quad x < -1$	M1: factorization or $x = \frac{4 \pm \sqrt{196}}{10}$	
	(ii)	$y = 5x^2 - 4x + k \quad \text{----- (1)}$ $y = 6x + 4 \quad \text{----- (2)}$ $(1) = (2), \quad 5x^2 - 4x + k = 6x + 4$ $5x^2 - 10x + k - 4 = 0$ <p>Since line is tangent to curve <math>\rightarrow D = 0</math></p> $(-10)^2 - 4(5)(k - 4) = 0$ $100 - 20k + 80 = 0$ $20k = 180$ $k = 9$	M1: obtain an equation in terms of $x$ using simultaneous equation method  M1: formulate equation with discriminant = 0  A1	e.c.f. is given
3	(i)	Area of right-angled triangle $= \frac{1}{2} \left( 9 - 2\sqrt{8} \right) \left( 7 - \frac{1}{\sqrt{2}} \right)$ $= \frac{1}{2} \left( 63 - \frac{9}{\sqrt{2}} - 14\sqrt{8} + \frac{2\sqrt{8}}{\sqrt{2}} \right)$ $= \frac{1}{2} \left( 63 - \frac{9}{\sqrt{2}} - 28\sqrt{2} + 4 \right)$	M1: expansion	

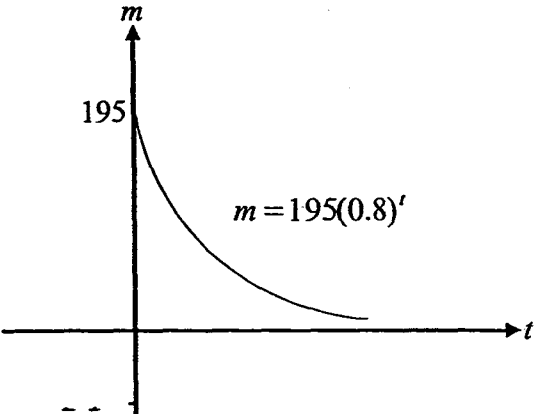
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		$= \frac{1}{2} \left( 67 - \frac{9 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} - 28\sqrt{2} \right)$ $= \frac{1}{2} \left( 67 - \frac{9}{2} \sqrt{2} - 28\sqrt{2} \right)$ $= \frac{1}{2} \left( 67 - \frac{9}{2} \sqrt{2} - 28\sqrt{2} \right)$ $= \frac{1}{2} \left( 67 - \frac{65}{2} \sqrt{2} \right)$ $= \frac{67}{2} - \frac{65}{4} \sqrt{2} \text{ cm}^2$	<p>M1: rationalizing denominator</p> <p>A1</p>	
		<p><u>Alternate method</u>            Area of right-angled triangle</p> $= \frac{1}{2} (9 - 2\sqrt{8}) \left( 7 - \frac{1}{\sqrt{2}} \right)$ $= \frac{1}{2} (9 - 2\sqrt{8}) \left( 7 - \frac{\sqrt{2}}{2} \right)$ $= \frac{1}{2} \left( 63 - \frac{9}{2} \sqrt{2} - 28\sqrt{2} + 4 \right)$ $= \frac{1}{2} \left( 67 - \frac{65\sqrt{2}}{2} \right)$ $= \frac{67}{2} - \frac{65}{4} \sqrt{2} \text{ cm}^2$	<p>M1: rationalize denominator</p> <p>M1: expansion</p> <p>A1</p>	
	(ii)	$BC^2$ $= (9 - 2\sqrt{8})^2 + \left( 7 - \frac{1}{\sqrt{2}} \right)^2$ $= 81 - 36\sqrt{8} + 32 + 49 - \frac{14}{\sqrt{2}} + \frac{1}{2}$ $= 81 - 72\sqrt{2} + 32 + 49 - \frac{14 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{1}{2}$ $= 81 - 72\sqrt{2} + 32 + 49 - 7\sqrt{2} + \frac{1}{2}$ $= \frac{325}{2} - 79\sqrt{2}$ <p><u>Alternate method</u>  <math display="block">BC^2</math> <math display="block">= (9 - 4\sqrt{2})^2 + \left( 7 - \frac{\sqrt{2}}{2} \right)^2</math> <math display="block">= 81 - 72\sqrt{2} + 32 + 49 - 7\sqrt{2} + \frac{1}{2}</math></p>	<p>M1: expansion</p> <p>M1: rationalizing denominator</p> <p>A1</p> <p>M1: rationalise denominator</p> <p>M1: expansion</p>	

		$= \frac{325}{2} - 79\sqrt{2}$	A1	
4	(i)	$75^{x-2} = 3^{2-x}$ $(3 \times 5^2)^{x-2} = 3^{2-x}$ $3^{x-2-2+x} \times 5^{2x-4} = 1$ $3^{2x-4} \times 5^{2x-4} = 1$ $15^{-4} \times 15^{2x} = 1$ $15^{2x} = 15^4$ $15^x = 15^2$ $15^x = 225 \text{ or } -225$ <p style="text-align: center;">(rejected since <math>15^x &gt; 0</math>)</p>	<p>M1: express 75 in its prime factors</p> <p>M1: simplify base 3 and 5</p> <p>M1 (same base)</p> <p>A1: negative must be rejected</p>	
	(ii)	$\left(5^{\frac{1}{2}}\right)^9 + \left(5^{\frac{1}{2}}\right)^7 + \left(5^{\frac{1}{2}}\right)^5 + \left(5^{\frac{1}{2}}\right)^3 - 155\left(5^{\frac{1}{2}}\right) = 5^p$ $625\left(5^{\frac{1}{2}}\right) + 125\left(5^{\frac{1}{2}}\right) + 25\left(5^{\frac{1}{2}}\right) + 5\left(5^{\frac{1}{2}}\right) - 155\left(5^{\frac{1}{2}}\right) = 5^p$ $625\left(5^{\frac{1}{2}}\right) = 5^p$ $5^4 \times 5^{\frac{1}{2}} = 5^p$ $5^{\frac{9}{2}} = 5^p$ <p>Comparing indices, <math>p = 4.5</math></p>	<p>M1: simplify the power</p> <p>M1: change to same base</p> <p>A1</p>	
5	(i)	$y = ab^x$ $\ln y = \ln ab^x$ $\ln y = \ln a + x \ln b$ $\ln a = 4.7$ $a = 109.9$ $a \approx 110$ $\frac{27.2 - 4.7}{9 - 0} = \ln b$ $b = 12.182$ $b \approx 12$ <p><u>Alternative Method</u></p> $\ln y = 2.5x + 4.7$ $y = e^{2.5x+4.7}$ $y = e^{4.7} \times e^{2.5x}$ <p>By comparing with <math>y = ab^x</math>,</p>	<p>M2: power law and quotient law</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	

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		$a = e^{4.7}$ $a \approx 110$ $b = e^{2.5}$ $b \approx 12$			
	(ii)	when $x = 2$ , $y = 109.9(12.182)^2$ $y \approx 16300(3s.f.)$  <u>Alternate method</u> Gradient = 2.5  $\ln y = 2.5x + 4.7$ $\ln y = 2.5(2) + 4.7$ $y = e^{9.7} \approx 16300$	M1: subst. truncated value A1	e.c.f is given provided exact or truncated value is used.	
6	(i)	(a)	when $t = 0$ , $m = 195(0.8)^0$ $m = 195(1)$ $m = 195$ gram	B1	
		(b)	when $t = 6$ , $m = 195(0.8)^6$ $m \approx 51.1(3s.f.)$ gram	B1	
		(c)	$\frac{1}{4}(195) = 195(0.8)^t$ $\frac{1}{4} = (0.8)^t$ $\ln\left(\frac{1}{4}\right) = \ln(0.8)^t$ $\ln\left(\frac{1}{4}\right) = t \ln(0.8)$ $t = \ln\left(\frac{1}{4}\right) \div \ln(0.8)$ $\approx 6.21$ years	M1: formulate equation. o.e.  M1: using natural log o.e.  M1: apply power law  A1	
	(ii)	Since the mass decreases with time, thus, the mass cannot never be more than 195 grams.  <u>Possible acceptable answer:</u> $t \rightarrow \infty$ , $0.8^t \rightarrow 0$ , $195(0.8^t) \rightarrow 0$ Therefore the mass can never be more than 195	B1: o.e.	Explain with respect to the context	

	(iii)		B1: general shape and interception with vertical axis shown clearly and correct	
7	(i)	<p>centre of circle <math>\hat{=} \begin{pmatrix} 8 &amp; -12 \\ -2 &amp; -2 \end{pmatrix}</math>  <math>= (-4, 6)</math></p> <p>Radius <math>= \sqrt{(-4)^2 + 6^2 - 27} = 5</math> units</p> <p><u>Alternate method:</u>  <math>x^2 + y^2 + 8x - 12y + 27 = 0</math>  <math>x^2 + 8x + 4^2 - 4^2 + y^2 - 12y + 6^2 - 6^2 + 27 = 0</math>  <math>(x + 4)^2 - 4^2 + (y - 6)^2 - 6^2 + 27 = 0</math>  <math>(x + 4)^2 + (y - 6)^2 = 25</math></p> <p>centre of circle <math>= (-4, 6)</math>          Radius <math>= 5</math> units</p>	B2: 1 mark for x and y coordinate each  B1	
	(ii)	<p>when <math>x = 0</math>, <math>y^2 - 12y + 27 = 0</math>  <math>(y - 9)(y - 3) = 0</math>  <math>y - 9 = 0</math> or <math>y - 3 = 0</math>  <math>y = 9</math> or <math>y = 3</math></p> <p>Centre of <math>C_2 = \left(0, \frac{9+3}{2}\right)</math>  <math>= (0, 6)</math></p> <p>Radius <math>= 3</math> units          Equation of <math>C_2</math> is <math>x^2 + (y - 6)^2 = 3^2</math></p>	M1  M1 A1	Accept: $(x - 0)^2 + (y - 6)^2 = 3^2$
	(iii)	<p>Circle 2:          when <math>x = -3</math> and <math>y = 3</math>, <math>(-3)^2 + (3 - 6)^2 = 18</math>,          which is more than <math>3^2</math> but less than <math>5^2</math>.</p> <p>Therefore the point <math>(-3, 3)</math> lies in <math>C_1</math> only.</p>	M1: value of 18 for comparison  A1: explain the comparison of $r^2$ value for both circles and conclude	

		<p><u>Alternate method</u></p> <p>Circle 1: radius = 5 units                      Circle 2: radius = 3 units                      Distance from centre of Circle 1 to point  <math>= \sqrt{(-4+3)^2 + (6-3)^2}</math>  <math>= \sqrt{10}</math> units</p> <p>Since <math>3 &lt; \sqrt{10} &lt; 5</math>, it lies within circle 1.</p>	<p>M1</p> <p>A1</p>	
8	(a)	<p>let <math>f(x) = 2x^3 + 3x^2 - 11x - 6</math>  <math>f(2) = 2(2)^3 + 3(2)^2 - 11(2) - 6 = 0</math>  <math>x - 2</math> is a factor of <math>f(x)</math>.</p> <p><math>2x^3 + 3x^2 - 11x - 6 = (x - 2)(2x^2 + bx + 3)</math>                      Comparing coeff. of <math>x</math>, <math>-11 = -2b + 3</math>  <math>b = 7</math></p> <p><math>2x^3 + 3x^2 - 11x - 6 = (x - 2)(2x^2 + 7x + 3)</math>  <math>= (x - 2)(2x + 1)(x + 3)</math></p> <p><math>2x^3 + 3x^2 - 11x - 6 = 0</math>  <math>(x - 2)(2x + 1)(x + 3) = 0</math>  <math>x - 2 = 0</math> or <math>2x + 1 = 0</math> or <math>x + 3 = 0</math>  <math>x = 2</math>                      <math>x = -\frac{1}{2}</math>                      <math>x = -3</math></p>	<p>M1: finding 1<sup>st</sup> factor</p> <p>M1: obtain quadratic form                      M1: factorise completely</p> <p>A2, minus 1 mark for 1 mistake</p>	
	(b)	<p><math>\frac{2x+3}{(x+1)(x^2-5)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-5}</math></p> <p><math>2x+3 = A(x^2-5) + (Bx+C)(x+1)</math></p> <p>Subst. <math>x = -1</math>, <math>2(-1)+3 = -4A</math>  <math>1 = -4A</math>  <math>A = -\frac{1}{4}</math></p> <p>Subst. <math>x = 0</math>, <math>3 = -\frac{1}{4}(-5) + (C)(-0+1)</math>  <math>3 = -\frac{1}{4}(-5) + (C)(1)</math>  <math>C = 1\frac{3}{4}</math></p>	<p>M1</p> <p>M1</p> <p>M1</p>	<p>For values of <math>A</math>, <math>B</math> and <math>C</math>, accept improper fraction or decimals</p>

			<p>Compare coeff. of <math>x^2</math>, <math>0 = -\frac{1}{4} + B</math></p> $B = \frac{1}{4}$ <p>therefore, <math>\frac{2x+3}{(x+1)(x^2-5)} = -\frac{1}{4(x+1)} + \frac{x+7}{4(x^2-5)}</math></p>	<p>M1</p> <p>A1</p>	
9	(i)	(a)	<p><math>\tan \theta = -1</math>  <math>\theta</math> lies in 2<sup>nd</sup> quadrant                      Basic angle, <math>\alpha = \tan^{-1} 1</math>  <math>= 45^\circ</math></p> <p>Therefore, <math>\theta = 180^\circ - 45^\circ</math>  <math>= 135^\circ</math></p>	<p>M1</p> <p>A1</p>	
		(b)	<p><math>y^2 = 9 - 4x</math> ----- (1)  <math>y = 1 - x</math> ----- (2)                      Subst. (2) into (1), <math>(1-x)^2 = 9 - 4x</math>  <math>1 - 2x + x^2 = 9 - 4x</math></p> $x^2 + 2x - 8 = 0$ $(x+4)(x-2) = 0$ <p><math>x+4 = 0</math> or <math>x-2 = 0</math>  <math>x = -4</math>                      <math>x = 2</math></p> <p>Subst. <math>x = -4</math> into (2), <math>y = 5</math>                      Subst. <math>x = 2</math> into (2), <math>y = -1</math></p> <p>Coordinates of <math>A</math> is <math>(-4, 5)</math> and coordinates of <math>B</math> is <math>(2, -1)</math>.</p>	<p>M1: substitution</p> <p>M1: quadratic equation</p> <p>A2: must express in terms of coordinates, if not, minus 1 mark</p>	
		(c)	<p>area of triangle <math>ABC = \frac{1}{2} \begin{vmatrix} 2 &amp; -4 &amp; 0 &amp; 2 \\ -1 &amp; 5 &amp; -3 &amp; -1 \end{vmatrix}</math></p> $= \frac{1}{2} (10 + 12 - 4 + 6)$ $= 12 \text{ units}^2$	<p>B1</p>	
	(ii)		<p>Since <math>AT : TB</math> is <math>1 : 2</math>, coordinates of <math>T</math> is <math>(-2, 3)</math></p> <p>Gradient of perpendicular line = 1</p> $1 = \frac{y-3}{x+2}$ $y-3 = x+2$ $y = x+5$	<p>M1</p> <p>M1</p> <p>A1</p>	

10	(i)	$2\log_7 p = 3 + 2\log_p 7$ $2\log_7 p = 3 + \frac{2}{\log_7 p}$ <p>Let <math>u = \log_7 p</math>, <math>2u = 3 + \frac{2}{u}</math></p> $2u^2 = 3u + 2$ $2u^2 - 3u - 2 = 0$ $(2u + 1)(u - 2) = 0$ $2u + 1 = 0 \quad \text{or} \quad u - 2 = 0$ $u = -\frac{1}{2} \quad \quad \quad u = 2$ $\log_7 p = -\frac{1}{2} \quad \quad \log_7 p = 2$ $p = 7^{-0.5} \quad \quad \quad p = 7^2$ $p \approx 0.378 \quad \quad \quad p = 49$	<p>M1: change of base</p> <p>M1: subst. method</p> <p>M1: solving for quadratic</p> <p>A2: 1 mark each</p>	
	(ii) (a)	$\log_x 9y = \log_x 9 + \log_x y$ $= 2\log_x 3 + \frac{\log_3 y}{\log_3 x}$ $= 2\log_x 3 + \frac{\log_3 y}{\log_3 x}$ $= \frac{2}{a} + \frac{1}{a}(\log_3 y)$ $= \frac{2}{a} + \frac{1}{a} \left( \frac{\log_9 y}{\log_9 3} \right)$ $= \frac{2}{a} + \frac{2b}{a}$ $= \frac{2+2b}{a}$	<p>M1: product law</p> <p>M2: power law and change of base s.o.i.</p> <p>A1</p>	
		<p><i>Alternate method:</i></p> $\log_x 9y = \log_x 9 + \log_x y$ $= \frac{\log_3 9}{\log_3 x} + \frac{\log_9 y}{\log_9 x}$ $= \frac{2}{a} + \frac{b \log_3 9}{\log_3 x}$ $= \frac{2}{a} + \frac{2b}{a}$ $= \frac{2+2b}{a}$	<p>M1: product law</p> <p>M2</p> <p>A1</p>	



		(b)	$x = 3^a$ and $y = 9^b$  $x^3 y = 3^{3a} \times 9^b$ $= 3^{3a} \times 3^{2b}$ $= 3^{3a+2b}$	M1: change to index form  M1: change to same base A1	
11	(a)	(i)	$\alpha + \beta = -\frac{-5}{3} = \frac{5}{3}$  $\alpha\beta = \frac{1}{3}$  $\frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$ $= \frac{\left(\frac{5}{3}\right)^2 - 2\left(\frac{1}{3}\right)}{\frac{1}{3}}$ $= 6\frac{1}{3}$	M1: sum of roots  M1: product of roots     M1   A1	
		(ii)	$1 - \frac{\alpha}{\beta} + 1 - \frac{\beta}{\alpha} = 2 - \frac{\alpha^2}{\alpha\beta} - \frac{\beta^2}{\alpha\beta}$ $= 2 - \frac{\alpha^2 + \beta^2}{\alpha\beta}$ $= 2 - 6\frac{1}{3}$ $= -4\frac{1}{3}$  $\left(1 - \frac{\alpha}{\beta}\right)\left(1 - \frac{\beta}{\alpha}\right) = 1 - \frac{\beta}{\alpha} - \frac{\alpha}{\beta} + 1$ $= 2 - \frac{\alpha^2 + \beta^2}{\alpha\beta}$ $= 2 - 6\frac{1}{3}$ $= -4\frac{1}{3}$  Quadratic equation is $x^2 + \frac{13}{3}x - \frac{13}{3} = 0$	M1: simplify     M1    M1   A1	accept improper fraction

	(b)	(i)	$\tan 45^\circ(\sin 60^\circ + \cos 30^\circ)$ $= 1\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)$ $= \sqrt{3}$	M1: express in exact form A1	
		(ii)	$3 \tan A + \cos\left(\frac{\pi}{2} - A\right)$ $= 3\left(\frac{3}{4}\right) + \frac{3}{5}$ $= 2\frac{17}{20}$	M1 A1	Calculator is allowed for this part, hence working involving use of calculator is acceptable.
12	(a)		$5 \operatorname{cosec} x + 11 = 0$ $\operatorname{cosec} x = -\frac{11}{5}$ $\sin x = -\frac{5}{11}$ $x$ lies in 3 <sup>rd</sup> and 4 <sup>th</sup> quadrant basic angle, $\alpha = \sin^{-1}\left(\frac{5}{11}\right)$ $x = \pi + \sin^{-1}\left(\frac{5}{11}\right)$ or $x = 2\pi - \sin^{-1}\left(\frac{5}{11}\right)$ $x \approx 3.61$ or $x \approx 5.81$	M1: simplify M1: identify basic angle A2	
	(b)	(i)	Period = 12 hours $\rightarrow \cos\left(\frac{\pi}{6}t\right)$ Therefore $k = \frac{\pi}{6}$ (shown)	B1: need to show period = 12 hr or some form of working.	Working: $k = \frac{2\pi}{12}$ $k = \frac{\pi}{6}$
		(ii)	Graph is shifted by 0.3 units upward Therefore $a = 1.65$ Amplitude of oscillation = $\frac{3-0.3}{2} = 1.35$ Therefore $b = 1.35$	B1 B1	