NAME: ______________________________ (   )   CLASS: ______

139

FAIRFIELD METHODIST SCHOOL (SECONDARY)

END-OF-YEAR EXAMINATION 2015
SECONDARY 3 EXPRESS

ADDITIONAL MATHEMATICS 4047

Date: 7 October 2015 **Duration: 2 hours 30 minutes**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Setters: Mr Joel Li and Mr Wilson Ho

This question paper consists of 7 printed pages including the cover page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 Express $\frac{5x-1}{x^2+x-12}$ in partial fractions. [3]

2 Given that $\sin\theta = \frac{3}{5}$, where θ is obtuse, evaluate each of the following.

(i) $\tan\theta$ [1]

(ii) $\cos(-\theta)$ [1]

(iii) $\sin(180° - \theta)$ [1]

3 Without using a calculator, solve the equation $\frac{2^{x+3}}{8^{-x}} = \frac{4^{\frac{1}{2}x}}{64}$. [4]

4 The function f is defined by $\mathrm{f}(x) = a\sin(bx) + c$ for $0 \le x \le \pi$, where a, b and c are positive integers. It is given that the amplitude of $\mathrm{f}(x)$ is 5 and that the period of $\mathrm{f}(x)$ is $\frac{2\pi}{3}$.

(i) State the value of a and of b. [2]

Given that the minimum value of $\mathrm{f}(x)$ is –2,

(ii) state the value of c, [1]

(iii) sketch the graph of $\mathrm{f}(x)$ for $0 \le x \le \pi$. [2]

5 **(i)** Simplify $\sqrt{52} + 2\sqrt{208} - \sqrt{117}$ and leave your answer in the simplest surd form. [2]

(ii) A rectangular container has a square base. The length of each side of the base is $(2\sqrt{5} - \sqrt{3})$ m and the volume of the container is $(45\sqrt{5} - 14\sqrt{3})$ m^3. Find, **without using a calculator**, the exact height of the container in the form $(a\sqrt{3} + b\sqrt{5})$ m, where a and b are integers. [4]

6 **(i)** Determine the set of values of m for which the equation $2x^2+4x+2m=6mx-2$ has no real roots. [4]

(ii) Hence, giving a reason, state what can be deduced about the curve $y=2(x+1)^2$ and the line $y=6x-2$. [2]

7 The remainder when $2x^3+4x^2-15x+12$ is divided by $x+a$ is three times the remainder when it is divided by $x-a$.

(i) Show that $2a^3+2a^2-15a+6=0$. [3]

(ii) Solve this equation completely. [4]

8 The roots of the quadratic equation $-2x^2+3x+8=0$ are α and β, find the quadratic equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [7]

9 **(i)** Solve the equation $2\tan x=3\sin x$, for $0°\le x\le 360°$, [4]

(ii) Given that the values of y is between 0 and 4, find the values for which $\cos(2y-1.5)=0.4$. [4]

10 A cup of hot liquid was put on the table to cool at 6.20 pm. The temperature of the water, T °C after x minutes is given by the formula $T = 20 + ne^{-mx}$, where m and n are constants. The table below shows the measured values of T and x.

x (minutes)	5	10	15	20	25
T (°C)	77.7	57.0	43.7	35.2	29.7

(i) Plot $\ln(T-20)$ against x and draw a straight line graph. [3]

Use your graph to estimate

(ii) the value of m and the value of n, [2]

(iii) the temperature of the liquid at 6.34 pm, [2]

(iv) the number of minutes it will take for the temperature of the liquid to drop to half of its initial value. [2]

11 **(i)** Given that $\log_p a = r$ and $\log_p b = s$, express $\log_{ab} p$ in terms of r and s [2]

(ii) Solve the equation $3e^x + 11e^{\frac{1}{2}x} = 20$. [4]

(iii) The population of a certain bacteria, P, present at time t hours after being observed initially is given by the formula $P = 480 + 300e^{0.5t}$.

(a) Calculate the initial population of the bacteria. [1]

(b) Find the population of the bacteria 12 hours after being observed initially and leave your answer correct to the nearest thousand. [1]

(c) Calculate the time taken for the bacteria to reach a population of 2700. Leave your answer correct to the nearest hour. [2]

12 **(i)** Solve the equation $|x^2 + 3x - 4| = 6$: [4]

(ii) Sketch the graph of $y = |x^2 + 3x - 4|$ for $-5 \le x \le 2$, indicating on your sketch, the x- and y-intercepts, and the turning point. [4]

(iii) Hence, find the solution(s) for which $|x^2 + 3x - 4| \le 6$. [2]

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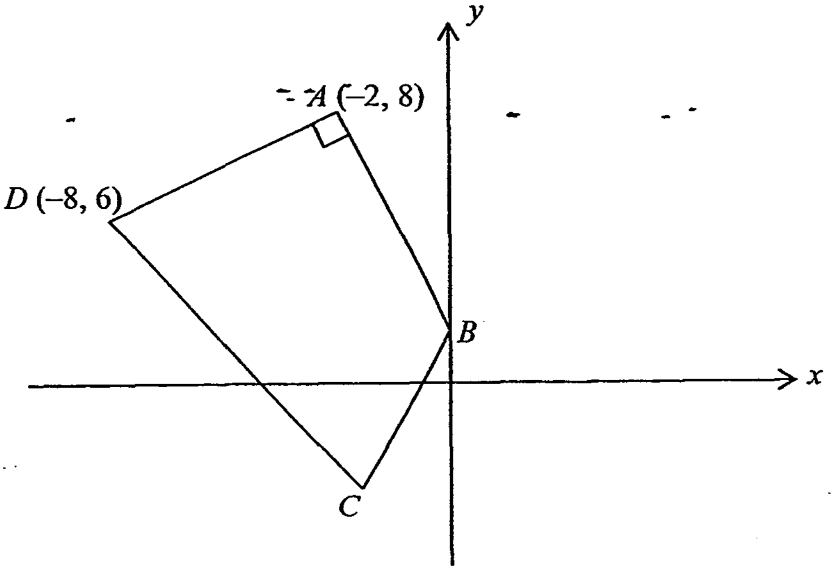
13 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a quadrilateral *ABCD* in which *A* is $(-2, 8)$ and *D* is $(-8, 6)$.

The point *C* lies on the perpendicular bisector of *AD* and the point *B* lies on the *y*-axis.

The equation of *CD* is $3y = -4x - 14$ and angle $DAB = 90°$.



(i) Find

(a) the equation of *AB*, [3]

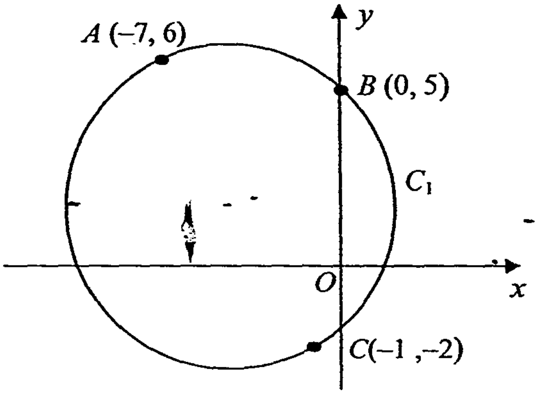
(b) the coordinates of *B*, [1]

(c) the equation of the perpendicular bisector of *AD*, [3]

(ii) Show the coordinates of *C* is $(-2, -2)$. [2]

(iii) Find the area of the quadrilateral *ABCD*. [2]

14 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, which is not drawn to scale, A, B and C are points on the circle C_1.

(i) Show that AC is the diameter of the circle C_1 and hence find the centre of C_1. [4]

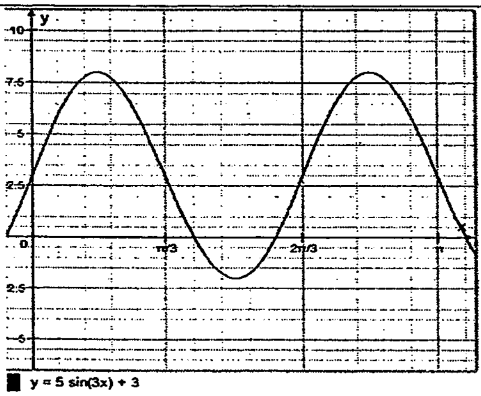
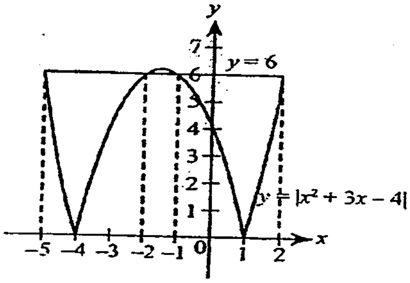
(ii) Find the equation of C_1 in the form $x^2+y^2+px+qy+r=0$, where p, q and r are integers. [3]

(iii) Given that C_2 is a reflection of the circle C_1 in the line $x=2$, find the centre of C_2 and the equation of C_2. [2]

(iv) Determine whether C lies inside or outside the circle C_2. [2]

~ End of Paper ~

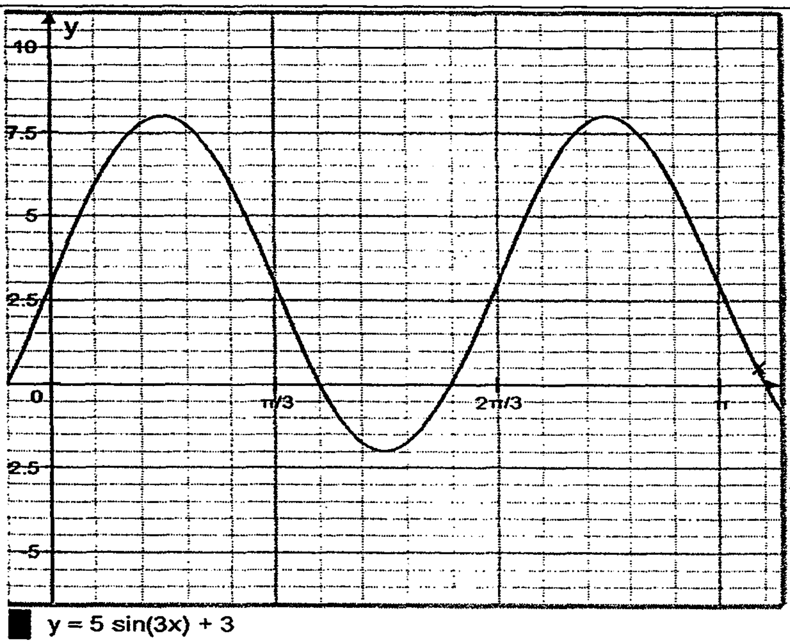
<table>
<tr><td></td><td>Sec 3 Add. Mathematics End-of-Year Exam 2015 Answer sheet</td><td></td><td></td></tr>
<tr><td>1</td><td>$\frac{5x-1}{x^2+x-12}=\frac{2}{(x-3)}+\frac{3}{(x+4)}$</td><td>10</td><td>
<table><tr><td>Ln(T-20)</td><td>4.06</td><td>3.61</td><td>3.17</td><td>2.72</td><td>2.27</td></tr></table>
(i) Correct points and axes
Line of best fit
(ii) $\ln(T-20)=\ln n-mx$ Hence gradient = − <i>m</i>,
vertical-intercept $=\ln n$
From the graph,
Gradient = $\frac{4.06-3.5}{5-11.25}\approx -0.0896$
<i>m</i> = 0.0896 [accept 0.080 to 0.090]
vertical-intercept $=\ln n = 4.5$
<i>n</i> ≈ 90.0 [accept 81.4 to 99.5]
(iii) When <i>x</i> = 14, $\ln(T-20)=3.25$
$\therefore T=e^{3.25}+20\approx 45.8°\text{C}$
[accept 44.5 to 47.1]
(iv) When <i>x</i> = 0, $\ln(T-20)=4.5$
$\therefore \text{Initial } T=110.0°\text{C}$
For $T=55.0°\text{C}$, $\ln(T-20)=3.56$
<i>x</i> = 10.5 min[accept 10 to 11.3]</td></tr>
<tr><td>2</td><td>(i) $-\frac{3}{4}$ (ii) $-\frac{4}{5}$ (iii) $\frac{3}{5}$</td><td>11</td><td>(i) $\log_{ab}p=\frac{\log_p p}{\log_p ab}=\frac{1}{\log_p a+\log_p b}$
$=\frac{1}{r+s}$</td></tr>
<tr><td>3</td><td><i>x</i> = -3</td><td></td><td>(ii) $3e^{2\left(\frac{x}{2}\right)}+11e^{\left(\frac{x}{2}\right)}=20$
Let $y=e^{\frac{x}{2}}$
$3y^2+11y-20=0$
$(y+5)(3y-4)=0$
$y=-5$ or $y=\frac{4}{3}$
$e^{\frac{x}{2}}=-5$ (reject)
Or $e^{\frac{x}{2}}=\frac{4}{3}$</td></tr>
</table>

			$\frac{x}{2}=\ln\frac{4}{3}$ $x=0.575(to3sf)$
4	(i) $a=5$ Period = $\frac{2\pi}{3}=\frac{2\pi}{b}$ $\therefore b=3$		(i) $p=480+300e^{0.5t}$ (a) t = 0, $p=480+300=780$ (b) $p=480+300e^{0.5(12)}$ = 121508.638= 122000(to the nearest thousand)
	(ii) $c=3$		(c) $2700=480+300e^{0.5t}$ $t=\frac{\ln 7.4}{0.5}=4.00296=4hours$
	(iii) Correct shape, accurate Max & Min points, *y*-intercept (y =8, y = -2, y= 3)		
		12	(i) $x=-5,-2,-1$ or 2 (ii) 
5	(i) $7\sqrt{13}$		(iii) From the graph, for $\left\|x^2+3x-4\right\|\le 6$, the answer is $-5\le x\le -2$ or $-1\le x\le 2$.
	(ii) Height= $2\sqrt{3}+3\sqrt{5}$	13	(i) (a)Equation of *AB*; $y=-3x+2$ (b) *B*(0, 2) (c) Eqn of perpendicular bisector of AD: $y=-3x-8$ (ii) $y=-3x-8$ --- (1) $3y=-4x-14$ --- (2) Solve simultaneous eqns; C(-2,-2) (iii) Line AC is parallel to y-axis. Area of ABC $=\frac{1}{2}(10)(2)$ $=10units^2(shown)$ Area of Quadrilateral ABCD $=10+\frac{1}{2}(10)(6)$ $=40units^2$

6	(i) $0<m<\frac{16}{9}$	14	(i) Gradient of *AB* = $\frac{6-5}{-7-0}=\frac{1}{-7}=-\frac{1}{7}$ Gradient of *BC* = $\frac{-2-5}{-1-0}=\frac{-7}{-1}=7$ Since GradAB x GradBC = $-\frac{1}{7}\times 7=-1$ Therefore AB is perpendicular to BC Therefore $\angle ABC=90°$ ($\angle$ in semi-circle) Therefore AC is the diameter of C_1 Centre of C_1 = $\left(\frac{-7-1}{2},\frac{6-2}{2}\right)$ $=(-4, 2)$
	(ii) $y=2(x+1)^2$ $y=6x-2$ (m=1 in part(i)) $=2(x+1)(x+1)$ $=2(x^2+2x+1)$ $=2x^2+4x+2$ (implies m = 1 in part (i))		(ii) Radius of C_1 = $\sqrt{(-7-(-4))^2+(6-(2))^2}$ $=\sqrt{25}=5units$ Eqn of C_1= $(x+4)^2+(y-2)^2=(5)^2$ $x^2+y^2+8x-4y-5=0$
	Since **m = 1** satisfies $0<m<\frac{16}{9}$, it implies that there is no real roots and therefore the curve and the line do not cut each other.		(iv) Distance of C from centre of C_2= $\sqrt{(-1-8)^2+(-2-2)^2}=\sqrt{97}$ Since $\sqrt{97}>\sqrt{25}$, C lies outside C_2
7	(i) $2(-a)^3+4(-a)^2-15(-a)+12$ $=3(2(a)^3+4(a)^2-15(a)+12)$ and simplify to show $2a^3+2a^2-15a+6=0$		
	(ii) Use trial and error to find factor (a-2) and long division to get $(a-2)(2a^2+6a-3)=0$ $\therefore\ a=2$ or $a=\frac{-6\pm\sqrt{36-4(2)(-3)}}{2(2)}$ $a=0.436$ or $a=-3.44$ (to 3 sf)		
8	Sum of roots = $-\frac{b}{a}=-\frac{3}{(-2)}=\frac{3}{2}=\alpha+\beta$ Product of roots = $\frac{c}{a}=\frac{8}{(-2)}=-4=\alpha\beta$		

	Sum of roots = $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$ = $-\frac{171}{512}$ Product of roots = $\frac{1}{-64}$		
	Equation $x^2-(\frac{-171}{512})x-\frac{1}{64}=0$ $x^2+\frac{171}{512}x-\frac{1}{64}=0$ or $512x^2+171x-8=0$		
9	(i) $x=0°,180°,360°$ $,x=$ $48.2°,311.8°$		
	(ii) $y=0.171$ $y=1.33,3.31$		

	Sec 3 Additional Mathematics End-of-Year Exam 2015	
Qn. No.	**Solution**	Allocation of marks
1	$\frac{5x-1}{x^2+x-12}=\frac{5x-1}{(x-3)(x+4)}$	
	$=\frac{A}{(x-3)}+\frac{B}{(x+4)}$	M1
	$=\frac{A(x+4)+B(x-3)}{(x-3)(x+4)}$	
	$A(x+4)+B(x-3)=5x-1$	
	Let x = -4, B(-7) = 5(-4) − 1	
	−7B = −21	
	B = 3	A1 for either getting A or B correct
	Let x = 3, 7A = 5(3) − 1	
	7A = 14	
	A = 2	
	$\frac{5x-1}{x^2+x-12}=\frac{2}{(x-3)}+\frac{3}{(x+4)}$	A1-must write this statement
2	(i) $-\frac{3}{4}$	B1
	(ii) $-\frac{4}{5}$	B1
	(iii) $\frac{3}{5}$	B1
3	$\frac{2^x2^3}{(2^3)^{(-x)}}=\frac{(2^2)^{\frac{1}{2}x}}{2^6}$	M1 – for expressing in Base 2
	$2^{4x}2^3=\frac{2^x}{2^6}$ **or** $2^{x+3+3x}=2^{x-6}$	
	$\frac{2^{4x}}{2^x}=\frac{1}{2^9}$ **or** $4x+3=x-6$	M1 – for manipulating and simplifying
	$2^{3x}=2^{-9}$ **or** $3x=-9$ **(M1)(for alternative method)**	
	$3x$ = -9	M1
	x = -3	A1

Qn. No.	Solution	Allocation of marks
4	(i) $a = 5$ Period = $\frac{2\pi}{3} = \frac{2\pi}{b} \quad \therefore b = 3$	B1 B1
	(ii) $c = 3$	B1
	(iii) Correct shape Accurate Max & Min points, y-intercept (y =8, y = -2, y=3).	C1 P1
		
5	(i) $\sqrt{52} + 2\sqrt{208} - \sqrt{117}$ $= \sqrt{4*13} + 2\sqrt{4*4*13} - \sqrt{9*13}$ $= 2\sqrt{13} + 8\sqrt{13} - 3\sqrt{13}$ $= 7\sqrt{13}$	M1 show ability to simplify surds(2nd step) A1
	(ii) Area = $\left(2\sqrt{5} - \sqrt{3}\right)^2$	
	$=\left(2\sqrt{5} - \sqrt{3}\right)\left(2\sqrt{5} - \sqrt{3}\right)$	
	$=20 - 2\sqrt{15} - 2\sqrt{15} + 3$	
	$=23 - 4\sqrt{15}$	B1 for calculating area in surd form
	Volume = $45\sqrt{5} - 14\sqrt{3}$	

Qn. No.	Solution	Allocation of marks
	Height $= \dfrac{45\sqrt{5}-14\sqrt{3}}{23-4\sqrt{15}} \times \dfrac{23+4\sqrt{15}}{23+4\sqrt{15}}$	M1 for rationalising surds
	$= \dfrac{1035\sqrt{5}+180\sqrt{75}-322\sqrt{3}-56\sqrt{45}}{529-240}$	
	$= \dfrac{1035\sqrt{5}+900\sqrt{3}-322\sqrt{3}-168\sqrt{5}}{289}$	M1 for expansion & simplifying
	$= \dfrac{578\sqrt{3}+867\sqrt{5}}{289}$	
	$= 2\sqrt{3}+3\sqrt{5}$	A1
6	(i) $2x^2+4x+2m=6mx-2$	
	$2x^2+4x+2m-6mx+2=0$	
	$2x^2+(4-6m)x+2m+2=0$	
	No real roots when $b^2-4ac<0$	
	$(4-6m)^2-4(2)(2m+2)<0$	M1
	$16-48m+36m^2-16m-16<0$	
	$36m^2-64m<0$	
	$9m^2-16m<0$	
	$m(9m-16)<0$	M1
	$m<0 \quad 9m-16>0$ or $m>0 \quad 9m-16<0$ $m<0 \quad 9m>16$ or $m>0 \quad 9m<16$ $m>\dfrac{16}{9}$ or $m<\dfrac{16}{9}$	M1
	$\therefore \ 0<m<\dfrac{16}{9}$	A1
	(ii) $y=2(x+1)^2$ $\quad y=6x-2$ (m=1 in part(i)) $=2(x+1)(x+1)$ $=2(x^2+2x+1)$ $=2x^2+4x+2$ (implies m = 1 in part (i))	M1—ability to deduce m = 1
	Since m = 1 satisfies $0<m<\dfrac{16}{9}$, it implies that there is no real roots and therefore the curve and the line do not cut each other.	A1

Qn. No.	Solution	Allocation of marks
7	(i) $2(-a)^3+4(-a)^2-15(-a)+12=3(2(a)^3+4(a)^2-15(a)+12)$	M1 for applying remainder thm and equating equations
	$-2a^3+4a^2+15a+12=3(2a^3+4a^2-15a+12)$	
	$-2a^3+4a^2+15a+12=6a^3+12a^2-45a+36$	
	$-8a^3-8a^2+60a-24=0$	M1 for simplifying
	$8a^3+8a^2-60a+24=0$	
	$2a^3+2a^2-15a+6=0$ (shown)	AG1 for dividing the equation by 4 and writing the statement
	(ii) Try a = 2 $2(2)^3+2(2)^2-15(2)+6$ $=16+8-30+6$ $=0$ $\therefore\ a-2$ is a factor	M1 for attempting to find a factor

Qn. No.	Solution	Allocation of marks
	Using long division, $\begin{array}{r} 2a^2+6a-3 \\ a-2\overline{)2a^3+2a^2-15a+6} \\ -(2a^3-4a^2) \\ \hline 6a^2-15a \\ -(6a^2-12a) \\ \hline -3a+6 \\ -3a+6 \\ \hline 0 \\ \hline \end{array}$	M1 using long division or other method to obtain other factors
	$2a^3+2a^2-15a+6=0$ $(a-2)(2a^2+6a-3)=0$ $\therefore\ a=2$ or $a=\dfrac{-6\pm\sqrt{36-4(2)(-3)}}{2(2)}$ $a=0.436$ or $a=-3.44$ (to 3 sf)	M1 for factorisation to obtain 1 more factor A1 for all solutions
8	Sum of roots = $-\dfrac{b}{a}=-\dfrac{3}{(-2)}=\dfrac{3}{2}=\alpha+\beta$	B1
	Product of roots = $\dfrac{c}{a}=\dfrac{8}{(-2)}=-4=\alpha\beta$	B1
	$\dfrac{1}{\alpha^3}+\dfrac{1}{\beta^3}=\dfrac{\beta^3+\alpha^3}{\alpha^3\beta^3}$ $=\dfrac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{(\alpha\beta)^3}$ $=\dfrac{(\alpha+\beta)([\alpha+\beta]^2-2\alpha\beta-\alpha\beta)}{(\alpha\beta)^3}$ $=\dfrac{(\alpha+\beta)([\alpha+\beta]^2-3\alpha\beta)}{(\alpha\beta)^3}$	M1 for using sum of cubes identity

	$=\dfrac{\frac{3}{2}([\frac{3}{2}]^2-3[-4])}{(-4)^3}$ $=\dfrac{\frac{3}{2}(\frac{9}{4}+12)}{-64}=-\dfrac{171}{512}$	A1
	Product of roots = $\left(\dfrac{1}{\alpha^3}\right)\left(\dfrac{1}{\beta^3}\right)$ $=\dfrac{1}{(\alpha\beta)^3}$ $=\dfrac{1}{(-4)^3}$ $=\dfrac{1}{-64}$	M1 B1
	Equation $x^2-(\dfrac{-171}{512})x-\dfrac{1}{64}=0$ $x^2+\dfrac{171}{512}x-\dfrac{1}{64}=0$ or $512x^2+171x-8=0$	A1
9	$2\tan x=3\sin x$ $\dfrac{2\sin x}{\cos x}=3\sin x$ (i) $2\sin x=3\sin x\cos x$ $2\sin x-3\sin x\cos x=0$ $\sin x(2-3\cos x)=0$ $\sin x=0 \qquad \cos x=\dfrac{2}{3}$ $x=0°,180°,360°$ $x=48.2°,311.8°$	M1 for applying identity M1 for factorising A1 A1

$\cos(2y-1.5)=0.4$

$2y-1.5=\cos^{-1}0.4$ M1

(ii)

$0<y<4$

$0<2y<8$

$-1.5<2y-1.5<6.5$ M1 (Show at least 1.159 and 5.124)

$2y-1.5=-1.159, 1.159, 5.124$

$2y=0.341, 2.659, 6.624$

$y=0.171$ A1

$y=1.33, 3.31$ A1

10

x (minutes)	5	10	15	20	25
T (°C)	77.7	57.0	43.7	35.2	29.7
$\ln(T-20)$	4.06	3.61	3.17	2.72	2.27

B1 for ln(T-20) values

(i) Correct points and axes P1

Line of best fit S1

(ii) $\ln(T-20)=\ln n-mx$ Hence gradient = − m, vertical-intercept $=\ln n$

From the graph,

Gradient = $\dfrac{4.06-3.5}{5-11.25}\approx-0.0896$

m = 0.0896 [accept 0.080 to 0.090] B1

vertical-intercept $=\ln n$ = 4.5

$n\approx$ 90.0 [accept 81.4 to 99.5] B1

(iii) When x = 14, $\ln(T-20)=3.25$ M1

$\therefore T=e^{3.25}+20\approx45.8°\text{C}$ [accept 44.5 to 47.1] A1

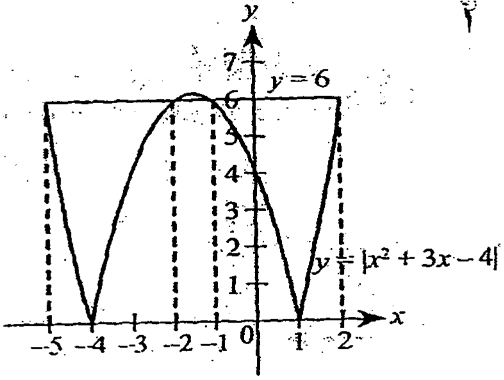
(iv) When x = 0, $\ln(T-20)=4.5$

$\therefore \text{Initial } T=110.0°\text{C}$ M1

For $T=55.0°\text{C}$, $\ln(T-20)=3.56$

x = 10.5 min [accept 10 to 11.3] A1

Qn. No.	Solution	Allocation of marks
11	(i) $\log_{ab} p = \dfrac{\log_p p}{\log_p ab}$ $= \dfrac{1}{\log_p a + \log_p b}$ $= \dfrac{1}{r+s}$	M1 for chang of base A1
	(ii) $3e^{2\left(\frac{x}{2}\right)} + 11e^{\left(\frac{x}{2}\right)} = 20$ Let $y = e^{\frac{x}{2}}$ $3y^2 + 11y - 20 = 0$ $(y+5)(3y-4) = 0$ $y = -5$ or $y = \dfrac{4}{3}$ $e^{\frac{x}{2}} = -5$ (reject) Or $e^{\frac{x}{2}} = \dfrac{4}{3}$ $\dfrac{x}{2} = \ln\dfrac{4}{3}$ $x = 0.575 (to 3sf)$	M1 for substitution and forming the quadratic eqn M1 factorising and solving A1 A1
	(iii) $p = 480 + 300e^{0.5t}$ (a) t = 0, $p = 480 + 300 = 780$ (b) $p = 480 + 300e^{0.5(12)}$ = 121508.638= 122000(to the nearest thousand)	B1 B1
	(c) $2700 = 480 + 300e^{0.5t}$ $e^{0.5t} = \dfrac{2220}{300}$ $0.5t = \ln 7.4$ $t = \dfrac{\ln 7.4}{0.5} = 4.00296 = 4 hours$	M1 A1

Qn. No.	Solution	Allocation of marks
12	(i) $x^2+3x-4=6$ or $x^2+3x-4=-6$ $x^2+3x-10=0$ $x^2+3x+2=0$ $(x-2)(x+5)=0$ $(x+1)(x+2)=0$ $x=2$ or -5 (A1) $x=-1$ or -2 (A1) $\therefore$ $x=-5,-2,-1$ or 2	M1 M1 for factorisation A1, A1
	(ii)  $y=x^2+3x-4$ $y=(x-1)(x+4)$ <u>y-intercept</u> $y=4$ <u>x-intercepts</u> $x=1$ or $x=-4$ <u>Line of symmetry</u> $x=-1.5$ <u>Max. point</u> $y=6.25$ Shape of curve (C1) Curve cuts y-intercept & x-intercepts (P1) Curve drawn within boundaries showing end-points (P1) & max. pt (P1) (iii) From the graph, for $\|x^2+3x-4\|\le 6$, the answer is $-5\le x\le -2$ or $-1\le x\le 2$.	A2
13	(i) Gradient of *AD* = $\dfrac{8-6}{-2-(-8)}=\dfrac{2}{6}=\dfrac{1}{3}$ Gradient of *AB* = − 3 Equation of *AB*; $y-8=-3(x+2)$ $y=-3x+2$ (ii) Sub $x=0$ into $y=-3x+2$, $y=2$ *B*(0, 2) (iii) Midpoint of AD $=\left(\dfrac{-2-8}{2},\dfrac{8+6}{2}\right)$ $=(-5,7)$	M1 B1 A1 B1 M1

	Gradient of perpendicular bisector of AD = – 3 Eqn of perpendicular bisector of AD: $y-7=-3(x+5)$ $y=-3x-15+7$ $y=-3x-8$ (iv) $y=-3x-8$ --- (1) $3y=-4x-14$ ---- (2) Solve simultaneous eqns; C(-2,-2) (v) Line AC is parallel to y-axis. Area of ABC $=\frac{1}{2}(10)(2)$ $=10 units^2 (shown)$ Area of Quadrilateral ABCD $=10+\frac{1}{2}(10)(6)$ $=40 units^2$	M1 A1 M1 A1 M1 / or applying shoelace method A1
14	(i) Gradient of $AB=\frac{6-5}{-7-0}=\frac{1}{-7}=-\frac{1}{7}$ Gradient of $BC=\frac{-2-5}{-1-0}=\frac{-7}{-1}=7$ Since GradAB x GradBC = $-\frac{1}{7}\times 7=-1$ Therefore AB is perpendicular to BC Therefore $\angle ABC=90°$ ($\angle$ in semi-circle) Therefore AC is the diameter of C_1 Centre of C_1 = $\left(\frac{-7-1}{2},\frac{6-2}{2}\right)$ = $(-4, 2)$	M1 for finding both gradient B1 for explanation and conclusion M1 A1
	(ii) Radius of C_1 = $\sqrt{(-7-(-4))^2+(6-(2))^2}$ = $\sqrt{25}=5units$ Eqn of C_1= $(x+4)^2+(y-2)^2=(5)^2$	M1 M1

	$x^2+y^2+8x-4y-5=0$	A1
	(iii) Centre of C_2 = (8 , 2), radius = 5 units Eqn of C_2= $(x-8)^2+(y-2)^2=25$	B1 B1
	(iv) Distance of C from centre of C_2=$\sqrt{(-1-8)^2+(-2-2)^2}=\sqrt{97}$ Since $\sqrt{97}>\sqrt{25}$, C lies outside C_2	M1 A1