Class	Register No	Name

Bukit Merah Secondary School
End of Year Examination 2015
Secondary 3 Express

E

ADDITIONAL MATHEMATICS 4047

2 October 2015

Additional Materials: Writing Paper (8 sheets)
Graph Paper (1 sheet)
Cover Page (1 sheet)

2 hours

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of the marks for this paper is **80**.

2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

1. It is given that the roots of $9x^2 - 18x + 11 = 0$ are $\alpha + 2\beta$ and $2\alpha + \beta$.

   (i) Show that $\alpha + \beta = \frac{2}{3}$. **[2]**

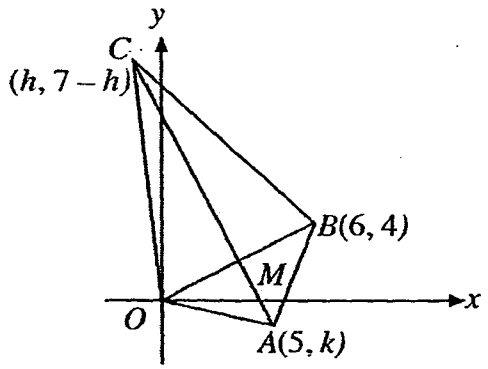
   (ii) Find a quadratic equation whose roots are α and β. **[4]**

2. The area of a rectangle is $\left(4\sqrt{2} + 9\right)$ cm^2 and its breadth is $\left(3 - \sqrt{2}\right)$ cm, find the perimeter of the rectangle in the form $\left(a + b\sqrt{2}\right)$ cm, where a and b are integers. **[3]**

3. **(a)** Find the range of values of x for which $x^2 > \frac{9x+5}{2}$. **[2]**

   (b) Find the least integer value of p for which the line $y = x + p$ does not intersect the curve $x^2 + xy + 2 = 0$. **[4]**

4. **The solution of this question by accurate drawing will not be accepted.**



The diagram, which is not drawn to scale, shows a kite $OABC$ whose diagonals meet at M. The coordinates of A, B and C are $(5, k)$, $(6, 4)$ and $(h, 7-h)$ respectively where h and k are constants. Find the

(i) coordinates of M, **[1]**

(ii) equation of AC, **[2]**

(iii) values of h and k, **[2]**

(iv) area of ΔOAB. **[2]**

5. **(a)** If $\sin\theta = -\frac{7}{25}$ and $\cos\theta < 0$, find the values of $\sec\theta$ and $\cot\theta$. **[4]**

   (b) Express $\frac{4x^3 + 5x^2 + x - 1}{x^2(x+1)}$ in partial fractions. **[5]**

[Turn Over

6. Given that $y = 2\sin 2x - 1$ is defined for all $0 \le x \le \pi$.

(i) State the period and amplitude of y. [2]

(ii) State the maximum and minimum values of y. [2]

(iii) Sketch the graph of y, indicating clearly all the answers to part (i) and (ii). [2]

(iv) On the diagram of part **(iii)**, sketch the graph of $y = 1 - \frac{x}{\pi}$ for $0 \le x \le \pi$. [1]

(v) **Hence,** state the number of solutions, for $0 \le x \le \pi$, of the equation $\pi - x = \pi(2\sin 2x - 1)$. [1]

7. (i) Write down the first three terms in the expansion, in ascending powers of x, of $(1 + 2x)^6$. [2]

(ii) If $(3 - x)(1 + 2x)^6 = 3 + ax + \frac{1}{2}bx^2 + \ldots$ find the values of a and b. [3]

8. **Answer the whole of this question on a piece of graph paper.**

Variables x and y are related by the equation $y = ax^3 + bx$ where a and b are constants. The table shows experimental values of x and y but an error has been made in recording one of the values of y.

x	2	3	4	5	6
y	14	36	76	125	234

(a) Using a scale of 2 cm to 5 units on each axis, plot $\frac{y}{x}$ against x^2 and draw a straight line. [3]

(b) Use your graph to correct the reading of y for which an error has been made. [2]

(c) Use your graph to estimate the value of a and of b. [3]

9. **(a)** ***Without using a calculator,*** evaluate $\log_3 2 \times \log_8 9$. [3]

(b) By using an appropriate substitution, solve the equation

$$3e^{2x} - e^x = 4.$$ [3]

10. **(a)** Prove that $(\sec x + \operatorname{cosec} x)(\sin x + \cos x) = \sec x \operatorname{cosec} x + 2$. [3]

(b) Find all the angles between 0° and 360° which satisfy the equation

$$2\cos\theta \sin\theta + \sin\theta = 0.$$ [3]

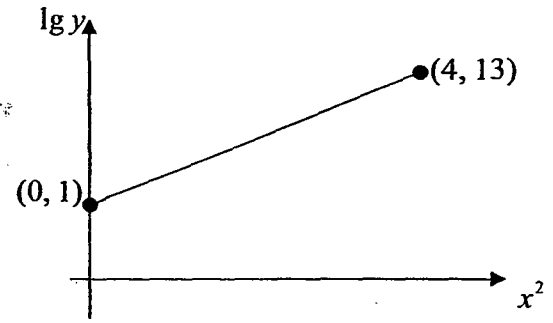
11. **(a)** Sketch the curve of $y = 3 - |2x - 5|$ for $-1 \le x \le 4$, indicating all the intercepts and the turning point clearly. [3]

(b) The expression $2x^3 + ax^2 + 3x + b$ where a and b are constants has a factor of $x - 1$ and leave a remainder of -54 when divided by $x + 2$.

(i) Find the value of a and of b. [3]

(ii) Factorise the expression $2x^3 + ax^2 + 3x + b$ completely. [3]

12. **(a)** The diagram shows part of a straight line drawn to represent the equation $py = 10^{qx^2}$, where p and q are constants. Find the values of p and of q. [3]



(b) The price of a new car on 1 January 2013 is \$100 000. Given that the value of the car will drop in such a way that, n months after the purchase, the sale price **\$P** is determined by the formula $P = 100000 \times e^{-0.015n}$.
Estimate the

(i) sale price of the car after 1 year, giving your answer correct to the nearest \$1000. [2]

(ii) month and year when the sale price of the car is less than \$50 000. [2]

End of Paper

BUKIT MERAH SECONDARY SCHOOL
2015 END-OF-YEAR EXAMINATION [Sec 3 Express]
ADDITIONAL MATHEMATICS (4047) – Answer Keys

1. (i) show that $\alpha+\beta=\frac{2}{3}$ (ii) $3x^2-2x+1=0$

2. Perimeter = $(4\sqrt{2}+16)$ cm

3. (a) $x<-\frac{1}{2}$ or $x\geq 5$ (b) $-4<p<4$, least integer value of p is -3.

4. (i) M(3, 2) (ii) Equation AC is $2y=-3x+13$
   (iii) h = -1, k = -1 (iv) Area = 13 unit^2.

5. (a) $\sec\vartheta=-\frac{25}{24}$, $\cot\vartheta=\frac{24}{7}$ (b) $4+\frac{2}{x}-\frac{1}{x^2}-\frac{1}{x+1}$

6. (i) Period = π, Amplitude = 2 (ii) max = 1, min = -3
   (v) Number of solutions = 2

7. (i) $1+12x+60x^2$ (ii) a = 35, b = 336

8. (a) $y=ax^3+bx\Rightarrow\frac{y}{x}=ax^2+b$, correct plot of all points, correct scales and label and smooth straight line drawn
   (b) correct reading of $\frac{y}{x}=28\Rightarrow\frac{y}{5}=28\Rightarrow y=140$
   (c) $a=grad=1$ and $b=3$

9. (a) $\frac{2}{3}$ (b) $\ln\frac{4}{3}$

10. (b) $\vartheta=120^0,180^0,240^0$

11. (b) a = -9, b = 4 and $f(x)=(x-1)(2x+1)(x-4)$

12. (a) q = 3, p = 0,1 (b) (i) \$84 000 (ii) Oct 2016

COVER PAGE

Class | Reg No

Candidate Name _____

BUKIT MERAH SECONDARY SCHOOL
END-OF-YEAR-EXAMINATION 2015
SECONDARY 3 EXPRESS

Subject Title/ Paper No :

Additional Mathematics

Instructions to Candidates:

1. Write the information required in the spaces provided above.
2. Write the number of questions attempted in the left hand margin of the writing paper and also in the left column of the table on this cover page.
3. Place this cover page on top of your answer sheets and tie them up using the string provided.
4. Answer sheets should not be tied so tightly that the examiners cannot turn the pages easily. Do not staple your answer sheets.

For Candidate's Use	For Examiner's Use
Question No	Marks Obtained
1	/6
2	/3
3	/6
4	/7
5	/9
6	/8
7	/5
8	/8
9	/6
10	/6
11	/9
12	/7
Total Marks	/80

Calculator Model: