BEDOK VIEW SECONDARY SCHOOL
END-OF-YEAR EXAMINATIONS 2015

CANDIDATE NAME

REGISTER NUMBER

CLASS

ADDITIONAL MATHEMATICS

4047

5 October 2015

2 hours

Additional Materials: Answer Paper

Secondary 3 Express

READ THESE INSTRUCTONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Setter: Ms Sally Ang

This document consists of **5** printed pages.

Do not turn over the page until you are told to do so.

2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta ABC = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 When the polynomial $g(x) = -x^3 + 4\frac{1}{2}x^2$ is divided by $x + 2n$, the remainder is $2 - 3n$.

(i) Show that $8n^3 + 18n^2 = 2 - 3n$. [3]

(ii) Solve $8n^3 + 18n^2 + 3n - 2 = 0$. [4]

(iii) Hence solve $\frac{64}{y^3} + \frac{72}{y^2} + \frac{6}{y} - 2 = 0$. [2]

2 Express $\frac{4 - x}{x^3 + 4x^2 + 4x}$ in partial fractions. [5]

3 Find the range of value of c for which $3x^2 + cx + 7 > 4$ for all values of x. [4]

4 It is given that $P(-1, -1)$ is the minimum point of the graph of $y = |-2x + k| - 1$.

(i) Show that the value of k is -2. [2]

(ii) Sketch the graph of $y = |-2x - 2| + k$, indicating the points of intersection with the coordinate axes. [3]

(iii) Hence write down the range(s) of values of x for which y is positive. [2]

5 **(i)** Find, in ascending powers of x, the first four terms of the expansion $(1 - 3x)^7$. [2]

(ii) Hence obtain the coefficient of x^2 in the expansion of $\left(2x + 3 - \frac{7}{5x}\right)(1 - 3x)^7$. [2]

6 Given that the constant term in the binomial expansion of $\left(x^2 - \frac{k}{2x}\right)^9$ is 5376, find the value of k. [4]

7 **(a)** Express $\frac{\sqrt{3} + 1}{3 - \sqrt{3}} + \frac{1}{2 + \sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are rational numbers. [5]

(b) If $p = \log_{10} 2$ and $q = \log_{10} 7$, express $\log_{10} \sqrt[3]{\frac{25}{49}}$ in terms of p and q. [3]

8 (a) Solve the equation $\log_4 y = 4 + 5\log_y 4$. [4]

(b) A student encounters the following question in an examination:

"Solve the equation $9^y + 5(3^y - 10) = 0$".

The following line shows the first step of the student's solution.

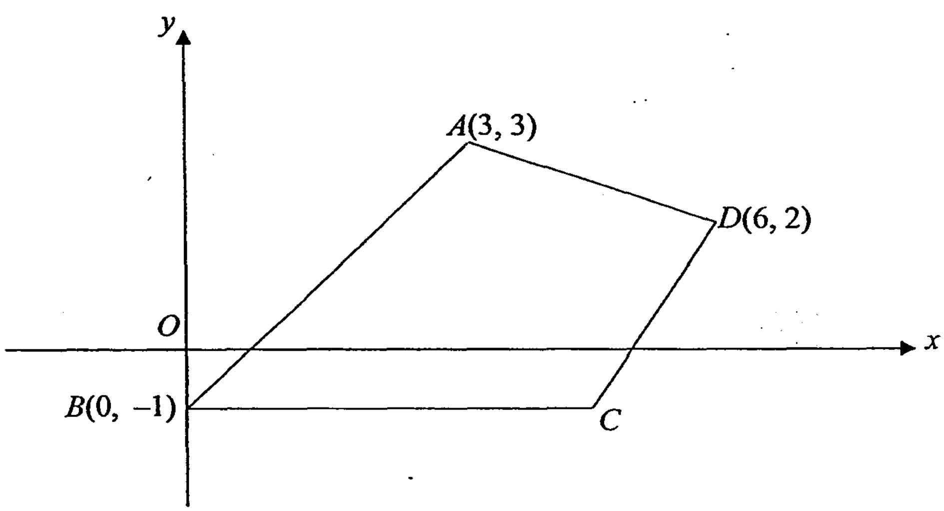
$$\lg 9^y + \lg 5(3^y) - \lg 50 = 0$$

State whether the student's first step is correct or wrong.

If it is correct, complete the steps for the student.

Otherwise, show the correct solution. [5]

9 The diagram below shows a kite *ABCD*, which is not drawn to scale. The coordinates of the points *A*, *B* and *D* are (3, 3), (0, – 1) and (6, 2) respectively.



(i) Find the equation of *AC*. [3]

(ii) Find the coordinates of *C*. [5]

(iii) What is the relationship between the *x*-axis and the equation of *BC*? [1]

10 Given that a circle *C*, with centre (3, 6), touches the *y*-axis,

(i) show that the equation of the circle is $x^2 + y^2 - 6x - 12y + 36 = 0$, [3]

(ii) find the equation of the circle C_1, which is a reflection of the circle *C* in the *y*-axis. [2]

11 (i) Sketch the graphs of $y = 2x^{\frac{1}{3}}$ and $y^2 = -2x$ on the same diagram. [2]

(ii) The two graphs $y = 2x^{\frac{1}{3}}$ and $y^2 = -2x$ intersect at (0, 0) and a point A. Find the coordinates of A. [3]

12 If $\cos\theta = -\frac{3}{5}$ and $\sin\theta > 0$, find the value of

(i) $\tan\theta + \sec(90° - \theta)$, [3]

(ii) $\cot\theta - \operatorname{cosec}\theta$. [3]

13 Without using a calculator, find the exact value and simplify each of the following into a single fraction where necessary:

(a) $\sin(-60°)$ [1]

(b) $\tan\frac{\pi}{6} + \sec\frac{\pi}{3}$ [2]

(c) $\cos\left(-\frac{10\pi}{3}\right)$ [2]

END OF PAPER

BEDOK VIEW SECONDARY SCHOOL

EOY EXAMINATIONS 2015
MARKING SCHEME

4047
5 October 2015
2 hours

ADDITIONAL MATHEMATICS
Secondary 3 Express

The total number of marks for this paper is 80.

Setters: Ms Sally Ang

3E A Math EOY Exam (4047) 2015
Marking Scheme

1(i)

$g(x) = -x^3 + 4\frac{1}{2}x^2$	
$g(-2n) = -(-2n)^3 + 4\frac{1}{2}(-2n)^2 = 2-3n$ [M1]	
$-(-8n^3) + 4\frac{1}{2}(4n^2) = 2-3n$ [M1]	
$8n^3 + 18n^2 = 2-3n$ (shown) [A1]	

1(ii)

Let $h(n) = 8n^3 + 18n^2 + 3n - 2$	
$h(-2) = 8(-2)^3 + 18(-2)^2 + 3(-2) - 2 = 0$ [M1]	
Hence, $(n+2)$ is a factor.	
$8n^3 + 18n^2 + 3n - 2 = (n+2)(8n^2 + An - 1)$ $18n^2 = An^2 + 16n^2$ $A = 2$ [M1]	Or long division
$8n^3 + 18n^2 + 3n - 2 = 0$	
$(n+2)(8n^2 + 2n - 1) = 0$	
$(n+2)(4n-1)(2n+1) = 0$ [M1]	
Hence, $(n+2)(4n-1)(2n+1) = 0$	
$n = -2, -\frac{1}{2}, \frac{1}{4}$ [A1]	For all answers

1(ii)

$\sin y = -2$ (NA)	
$= -\frac{1}{2}, \frac{1}{4}$ [M1]	
ref $\angle = 30°$ or $14.478°$ [M1]	
$y = 180° + 30°,\ 360° - 30°,\ 14.478°,\ 180° - 14.478°$	FT if ans from (ii) is wrong
$y = 14.5°$ or $165.5°$, $210°$ (NA) or $330°$ (NA) [A1$\sqrt{}$]	For all answers

Total marks = 10 marks

2

$\frac{4-x}{x^3+4x^2+4x}=\frac{4-x}{x(x+2)^2}$ $=\frac{A}{x}+\frac{B}{(x+2)}+\frac{C}{(x+2)^2}$ [M1] $4-x=A(x+2)^2+B(x)(x+2)+C(x)$ [M1] When $x=0$, $4=A(2)^2$ $A=1$ When $x=-2$ $6=-2C$ $C=-3$ When $x=1$, $3=9A+B(1)(3)+C(1)$ $3=9+3B-3$ $3B=-3$ $B=-1$ [M1] $\frac{4-x}{x^3+4x^2+4x}=\frac{1}{x}-\frac{1}{(x+2)}-\frac{3}{(x+2)^2}$ [A2]	Or other methods

Total marks = 5 marks

3E A Math EOY Exam (4047) 2015
Marking Scheme

3

$3x^2 + cx + 7 > 4$

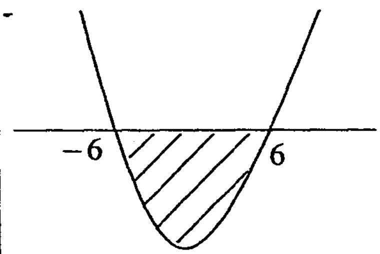
$3x^2 + cx + 3 > 0$ [M1]

$b^2 - 4ac < 0$

$c^2 - 4(3)(3) < 0$ [M1]

$c^2 - 6^2 < 0$

$(c+6)(c-6) < 0$ [M1]



$\therefore -6 < c < 6$ [A1]

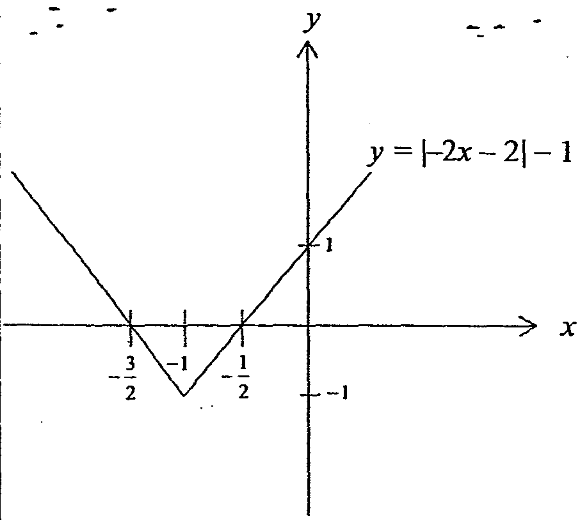
Total marks = 4 marks

3E A Math EOY Exam (4047) 2015
Marking Scheme

4(i)

$y=\lvert -2x+k\rvert-1$ Sub $(-1,\ -1)$: $-1=\lvert 2+k\rvert-1$ [M1] $2+k=0$ $k=-2$ (Shown) [A1]	

4(ii)

 B1 – shape of graph B1 – indicating x-intercepts B1 – indicating y-intercept	Deduct P each if labelling of graph and axes are not done. (must be symmetrical)

4(iii)

$x<-\frac{3}{2}$ or $x>-\frac{1}{2}$ [B2]	

Total = 7 marks

3E A Math EOY Exam (4047) 2015
Marking Scheme

5(i)

$(1-3x)^7 = 1+7(-3x)+\binom{7}{2}(-3x)^2+\binom{7}{3}(-3x)^3+\ldots$ [M1] $= 1-21x+189x^2-945x^3+\ldots$ [A1]	

5(ii)

$\left(2x+3-\frac{7}{5x}\right)(1-3x)^7$ $= \left(2x+3-\frac{7}{5x}\right)(1-21x+189x^2-945x^3+\ldots)$ $= \ldots.+2x(-21x)+3(189x^2)-\frac{7}{5x}(-945x^3)+\ldots$ [M1] $= 1848x^2+\ldots..$ Coefficient of $x^2 = 1848$ [A1]	For expansion of correct terms

Total = 4 marks

6

General term $= \binom{9}{r}(x^2)^{9-r}\left(-\frac{k}{2x}\right)^r$ $= \binom{9}{r}\left(-\frac{k}{2}\right)^r(x)^{18-2r}x^{-r}$ $= \binom{9}{r}\left(-\frac{k}{2}\right)^r(x)^{18-3r}$ [M1] $18-3r=0$ $r=6$ [A1] $\binom{9}{6}\left(-\frac{k}{2}\right)^6 = 5376$ [M1] $\frac{21}{16}k^6 = 5376$ $k^6 = 4096$ $k = 4$ [A1]	

Total = 4 marks

7(a)

$\frac{\sqrt{3}+1}{3-\sqrt{3}}+\frac{1}{2+\sqrt{3}}=\frac{(\sqrt{3}+1)(2+\sqrt{3})+(3-\sqrt{3})}{(3-\sqrt{3})(2+\sqrt{3})}$ [M1]	
$=\frac{(2\sqrt{3}+3+2+\sqrt{3})+3-\sqrt{3}}{6+3\sqrt{3}-2\sqrt{3}-3}$ [M1]	
$=\frac{2\sqrt{3}+8}{3+\sqrt{3}}\times\frac{3-\sqrt{3}}{3-\sqrt{3}}$ [M1]	
$=\frac{6\sqrt{3}-6+24-8\sqrt{3}}{9-3}$ [M1]	
$=\frac{18-2\sqrt{3}}{6}$	
$=3-\frac{1}{3}\sqrt{3}$ [A1]	

7(b)

$\log_{10}\sqrt[3]{\frac{25}{49}}=\log_{10}\left(\frac{25}{49}\right)^{\frac{1}{3}}$	
$=\frac{1}{3}[\log_{10}25-\log_{10}49]$ [M1]	Use of laws of log
$=\frac{1}{3}[2\log_{10}5-2\log_{10}7]$	
$=\frac{2}{3}\left[\log_{10}\frac{10}{2}-\log_{10}7\right]$ [M1]	5=10/2
$=\frac{2}{3}[\log_{10}10-\log_{10}2-q]$	
$=\frac{2}{3}[1-p-q]$ [A1]	

Total = 8 marks

3E A Math EOY Exam (4047) 2015
Marking Scheme

8(a)

$\log_4 y = 4 + 5\log_y 4$	
$\log_4 y = 4 + 5\left(\dfrac{\log_4 4}{\log_4 y}\right)$ [M1]	Change of base
Let $\log_4 y = x$,	
$x = 4 + \dfrac{5}{x}$	
$x^2 - 4x - 5 = 0$	
$(x-5)(x+1) = 0$	
$x = 5$ or $x = -1$	
$\log_4 y = 5$ $\quad\log_4 y = -1$ [M1]	Change to index form
$y = 4^5$ $\quad y = 4^{-1}$ [M1]	For both answers
$= 1024$ $\quad = \dfrac{1}{4}$ [A1]	

8(b)

The step is wrong. [B1]	
$9^y + 5(3^y - 10) = 0$	
$3^{2y} + 5(3^y - 10) = 0$	
Let $u = 3^y$,	
$u^2 + 5(u - 10) = 0$ [M1]	
$u^2 + 5u - 50 = 0$	
$(u+10)(u-5) = 0$	
$u = -10$ or $u = 5$	
$3^y = -10$ (rej) $\quad 3^y = 5$ [M1]	For both and reject −10
$y = \dfrac{\lg 5}{\lg 3}$ [M1]	Apply log
$= 1.46$ (3s.f) [A1]	

Total = 9 marks

9(i)

Eqn of $BD: y=\frac{1}{2}x-1$ $m_{AC}=-2$ [M1] $y-3=-2(x-3)$ [M1] Eqn of $AC: y=-2x+9$ [A1]	

9(ii)

Gradient $BD=\frac{2+1}{6-0}$ $=\frac{1}{2}$ Eqn of $BD: y=\frac{1}{2}x-1 \quad -(1)$ [B1] $y=-2x+9 \quad -(2)$ $-2x+9=\frac{1}{2}x-1$ [M1] $x=4, y=1$ midpoint of $AC=(4,1)$ [A1√]	FT if use wrong answer from (i)
Let point $C=(x,y)$ $(4,1)=\left(\frac{x+3}{2},\frac{y+3}{2}\right)$ [M1] $x=5, y=-1$ Coordinates of $C=(5,-1)$ [A1√]	FT if use wrong answer from (i)

9(iii)

The 2 lines are parallel. [B1]	Accept similar explanation

Total = 9 marks

3E A Math EOY Exam (4047) 2015
Marking Scheme

10(i)

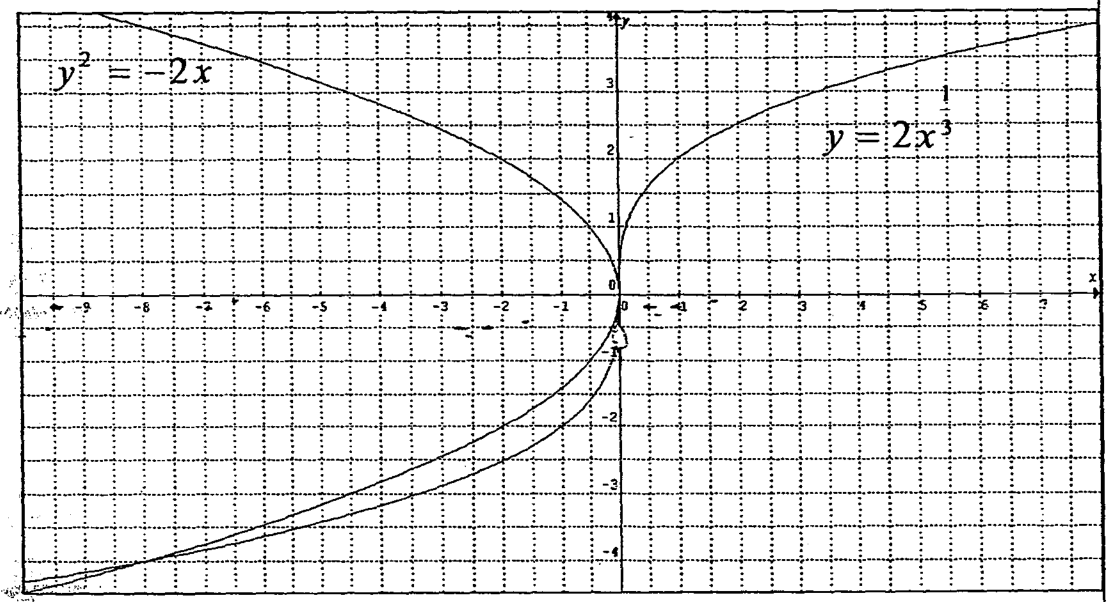
Centre = (3, 6), $r = 3$ [B1] Equation of circle C is $(x-3)^2 + (y-6)^2 = 3^2$ [M1] $x^2 - 6x + 9 + y^2 - 12y + 36 - 9 = 0$ $x^2 + y^2 - 6x - 12y + 36 = 0$ (shown) [A1]	

10(ii)

Centre = (–3, 6), $r = 3$ [B1] Equation of circle C_1 is $(x+3)^2 + (y-6)^2 = 3^2$ [B1]	Accept $x^2 + 6x + 9 + y^2 - 12y + 36 - 9 = 0$ $x^2 + y^2 + 6x - 12y + 36 = 0$ [B1]

Total = 5 marks

11(i)



B1 – Graph of $y = 2x^{\frac{1}{3}}$

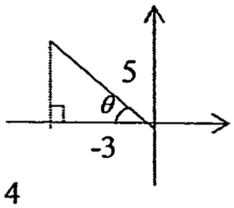
B1 – Graph of $y^2 = -2x$

11(ii)

$\left(2x^{\frac{1}{3}}\right)^2 = -2x$ [M1] $4x^{\frac{2}{3}} = -2x$ $x^{\frac{2}{3}} = -\frac{1}{2}x$ $x^2 = \left(-\frac{1}{2}x\right)^3$ $x^2 + \frac{1}{8}x^3 = 0$ [M1] $x^2\left(1 + \frac{1}{8}x\right) = 0$ $x = 0$ (NA) or -8 $y = -4$ Coordinates of $A = (-8, -4)$ [A1]	Accept other methods

Total = 5 marks

12(i)

Diagram [B1]  $\sin\theta = \frac{4}{5}$ $\tan\theta = -\frac{4}{3}$ [B1]	

12(ii)

$\cot\theta - \operatorname{cosec}\theta = \left(-\frac{3}{4}\right) - \left(\frac{5}{4}\right)$ [M1, M1] $= -2$ [A1]	

Total = 5 marks

13(a)

$\sin(-60°) = -\sin 60°$ $= -\frac{\sqrt{3}}{2}$ [B1]	

13(b)

$\tan\frac{\pi}{6} + \sec\frac{\pi}{3} = \tan\frac{\pi}{6} + \frac{1}{\cos\frac{\pi}{3}}$ $= \frac{\sqrt{3}}{3} + 2$ [M1] $= \frac{\sqrt{3}+6}{3}$ [A1]	To be able apply special angles' ratio

13(c)

$\cos\left(-\frac{10\pi}{3}\right) = -\cos\frac{\pi}{3}$ [M1] $= -\frac{1}{2}$ [A1]	

Total = 5 marks