BEATTY SECONDARY SCHOOL

END OF YEAR EXAMINATION 2015

SUBJECT : Additional Mathematics	**LEVEL : Secondary 3 Express**
PAPER : 4047/ 01	**DURATION : 2 hours**
SETTER : Mr Eric Koh	**DATE : 8 October 2015**

CLASS :	**NAME :**	**REG NO :**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Express $\frac{8-3\sqrt{2}}{4+3\sqrt{2}}$ in the form $a+b\sqrt{2}$, where a and b are integers. [3]

2 Without using a calculator, solve, for x and y, the simultaneous equations. [4]

$8^x \div 2^y = 64,$

$3^{4x} \times \left(\frac{1}{9}\right)^{y-1} = 81.$

3 Solve the following equations

(i) $1+\log_2 7 = 2\log_2(x-5)$ [3]

(ii) $\lg(7x-1)+\lg(x+2)=2$ [3]

4 α and β are the roots of the quadratic equation $2x^2-5x-6=0$.

Find

(i) the value of $\alpha^2+\beta^2$. [3]

(ii) the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. [3]

5 (i) Find the range of values of x for which $x(10-x) \geq 24$. [2]

(ii) Find the value of k for which the line $2y+x=k$ is a tangent to the curve $y^2+4x=20$.

[3]

(iii) Show that the equation $x^2+(2a-2)x+(2a-3)=0$ has real roots for all real values of a. [3]

6 Express the following algebraic fractions as a sum of partial fractions.

(i) $\frac{x+7}{(x-2)(x+1)}$ [3]

(ii) $\frac{x^2+2x+1}{x(x^2-4)}$ [3]

7 (i) Given the identity $x^4 + x^2 + x + 1 = (x^2 + A)(x^2 - 1) + Bx + C$, find the value of A, of B and of C. [4]

(ii) By substituting a suitable value for x in the identity, find the remainder when 100 010 101 is divided by 101. [2]

8 Sketch the graph of $y = |x - 2|$ for the domain $-3 \leq x \leq 3$. [3]

9 From the beginning of year 2010, the value of a precious stone increased continuously. The value of stone, \$$V$, after T years is given by the equation $V = 3000(4^T) + 1000(16^T)$.

Find

(i) the value of the precious stone at the beginning of year 2010, [1]

(ii) the shortest period of time for the value of the precious stone to reach \$10 000. [4]

10 (i) Expand fully $(1+2x)^4$ in ascending powers of x. [3]

(ii) In the expansion of $(2+x)^{10}$, the coefficient of x^9 is k. Find the value of k. [3]

(iii) Find the coefficient of x^{13} in the expansion $(1+2x)^4(2+x)^{10}$. [2]

11 Without using calculator, find the **exact** value of the following.

(i) $\dfrac{\sin 45^\circ}{\cos 30^\circ + \sin 60^\circ}$ [2]

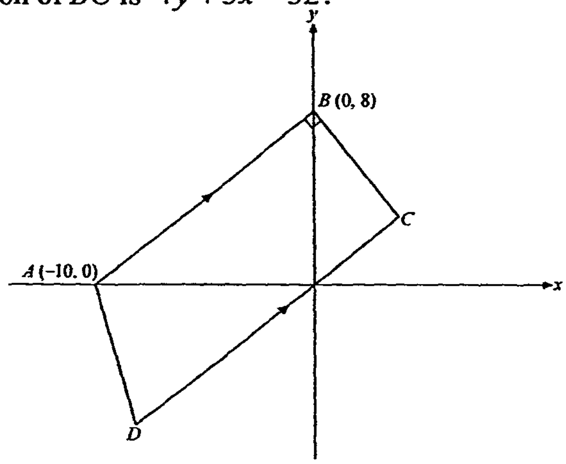
(ii) $\sin\dfrac{\pi}{3}\cos\dfrac{\pi}{6} + \cos\dfrac{\pi}{3}$ [2]

12 In a triangle ABC, angle A is a right angle. Given that $\sin B = x$, express, in terms of x,

(i) $\cos B$, [2]

(ii) $\tan B$. [2]

13 The diagram shows a trapezium $ABCD$ where the coordinates of A and B are (-10, 0) and (0, 8) respectively.
AB is parallel to DC, and AB is perpendicular to BC.
The equation of BC is $4y + 5x = 32$.



(i) Given that DC passes through origin O, find the equation of CD. [2]

(ii) Given that $\dfrac{\text{gradient of } AD}{\text{gradient of } AB} = -3$, find the equation of AD. [3]

(iii) Find the coordinates of the point C. [2]

(iv) Find the area of trapezium $ABCD$. [3]

14 **Answer the whole of this question on a sheet of graph paper.**

The table shows some experimental values of x and y.

x	1	2	3	4	5
y	2.25	2.40	2.50	2.57	2.63

It is known that x and y are related by an equation of the form $y = \dfrac{x}{ax+b} + 2$,
where a and b are constants.

(i) Plot $\dfrac{x}{y-2}$ against x for the given data to obtain a straight line graph. [3]

(ii) Use the graph to estimate the value of a and of b. [2]

(iii) Use your graph to estimate the value of x when $2(x + 9) = 9y$. [2]

-End Of Paper-

Beatty Secondary School

End of Year Examination 2015 **Additional Mathematics** **Sec 3 Express**

Marking Scheme

1.

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}$$
$$=\frac{8-3\sqrt{2}}{4+3\sqrt{2}}\times\frac{4-3\sqrt{2}}{4-3\sqrt{2}}$$ (M1)
$$=\frac{32-24\sqrt{2}-12\sqrt{2}+9(2)}{(4)^2-\left(3\sqrt{2}\right)^2}$$
$$=\frac{32-36\sqrt{2}+18}{16-18}$$
$$=\frac{50-36\sqrt{2}}{-2}$$ (A1)
$$=\frac{-2\left(-25+18\sqrt{2}\right)}{-2}$$
$$=-25+18\sqrt{2}$$ (A1)

2.

$$8^x \div 2^y = 64$$
$$\Rightarrow 2^{3x} \div 2^y = 2^6$$
$$\Rightarrow 2^{3x-y} = 2^6$$
$$\Rightarrow 3x-y=6-(1)$$ (M1)

$$3^{4x}\times\left(\frac{1}{9}\right)^{y-1}=81$$
$$\Rightarrow 3^{4x}\times\left(3^{-2}\right)^{y-1}=3^4$$
$$\Rightarrow 3^{4x-2y+2}=3^4$$
$$\Rightarrow 4x-2y+2=4$$
$$\Rightarrow 4x-2y=2-(2)$$ (M1)

Solving Equation (1) and (2)

$x=5,\ y=9$

(A1) (A1)

3.(i)

$1+\log_2 7 = 2\log_2(x-5)$

$\log_2 2+\log_2 7 = 2\log_2(x-5)$

$2\times 7 = (x-5)^2$

$x^2-10x+11=0$ M1

$x = \dfrac{-(-10)\pm\sqrt{(-10)^2-4(1)(11)}}{2(1)}$

$= 1.258$ *or* 8.742

$= 1.26$ *or* 8.74

A1 A1

(rejected)

3.(ii)

$\lg(7x-1)+\lg(x+2)=2$

$\lg(7x-1)(x+2)=\lg 100$

$(7x-1)(x+2)=100$

$7x^2+13x-2=100$ M1

$7x^2+13x-102=0$

$(x-3)(7x+34)=0$

$x=3$ or $x=-\dfrac{34}{7}$ (rejected)

A1

$\therefore x=3$

A1

4. (i)

$2x^2 - 5x - 6 = 0$

$\alpha + \beta = \frac{5}{2}$ B1

$\alpha\beta = -3$ B1

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (\frac{5}{2})^2 - 2(-3)$

$= 12\frac{1}{4}$ B1

4. (ii)

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$ B1

$= \frac{\frac{5}{2}}{-3}$

$= -\frac{5}{6}$

$(\frac{1}{\alpha})(\frac{1}{\beta}) = \frac{1}{\alpha\beta}$ B1

$= -\frac{1}{3}$

Equation

x^2 - (sum of roots) x + (product of roots) = 0

$x^2 - (-\frac{5}{6})x + (-\frac{1}{3}) = 0$

$x^2 + \frac{5}{6}x - \frac{1}{3} = 0$

$6x^2 + 5x - 2 = 0$ A1

5. (i)

$x(10-x) \geq 24$

$10x - x^2 \geq 24$ M1

$x^2 - 10x + 24 \leq 0$

$(x-4)(x-6) \leq 0$ A1

Ans : $4 \leq x \leq 6$

Alternative :

$(x-12)(x+2) \leq 0$

Ans : $-2 \leq x \leq 12$

5.(ii)

$y^2 + 4x = 20 ...(1)$

$2y + c = k$

$y = \dfrac{k-x}{2} ...(2)$

sub (2) into (1)

$\left(\dfrac{k-x}{2}\right)^2 + 4x = 20$

$\dfrac{(k-x)^2}{4} + 4x = 20$

$(k-x)^2 + 16x = 80$

$k^2 - 2kx + x^2 + 16x - 80 = 0$

$x^2 + (16-2k)x + k^2 - 80 = 0$ M1

Since the line is a tangent,

$b^2 - 4ac = 0$ A1

$(16-2k)^2 - 4(1)(k^2 - 80) = 0$

$-64k + 576 = 0$

$k = 9$ A1

5.(iii)

For real roots, $b^2 - 4ac \geq 0$

$(2a-2)^2 - 4(1)(2a-3)$ M1

$= 4a^2 - 8a + 4 - 8a + 12$

$= 4a^2 - 16a + 16$ A1

$= 4(a-2)^2$ i.e. always postive > 0 A1

Hence the equations has real roots.

6.(i)

$$\frac{x+7}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x+1)} \quad \text{M1}$$

$x+7 = A(x+1) + B(x-2)$

Let $x = -1$,

$B = -2$ A1

Let $x = 2$,

$A = 3$ A1

6.(ii)

$$\frac{x^2+2x+1}{x(x^2-4)} = \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x+2)} \quad \text{M1}$$

$x^2+2x+1 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$

Let $x = 2$,

$B = \frac{9}{8}$ A1

Let $x = 0$,

$A = -\frac{1}{4}$

Let $x = -2$,

$C = \frac{1}{8}$ A1

7.(i)

$$x^4 + x^2 + x + 1 \equiv (x^2 + A)(x^2 - 1) + Bx + C$$

$$x^4 + x^2 + x + 1 \equiv (x^2 + A)(x - 1)(x + 1) + Bx + C$$ M1

Let x = 1 | Let x = -1

$4 = B + C$ A1 A1 | $2 = -B + C$

$\therefore C = 3; B = 1$

Let x = 0

$$1 = (A)(-1)(1) + C$$
$$1 = -A + 3$$
$$A = 2$$ A1

7.(ii)

$100\,010\,101 = 1 + 100 + 10000 + 100000000$

$= (100)^4 + (100)^2 + (100) + 1$

$9999 = 10000 - 1 = (100)^2 - 1$

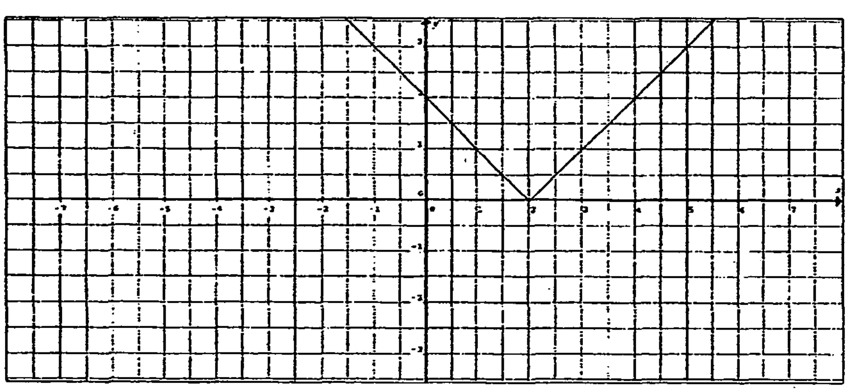
By letting x = 100,

$(100)^4 + (100)^2 + (100) + 1$ M1

$\equiv ((100)^2 + 2)(100 - 1)(100 + 1) + (100) + 3$

Remainder = 2 A1

8.



G3

Sketch the graph of $y = x - 2$ for $-3 \le x \le 3$ and reflect the part below the x-axis in the x-axis while retaining the part above the x-axis.

9.

(i)

Let $T=0$, $V=3000(4^0)+1000(16^0)$

$= 4000$ B1

(ii) Let $V = 10\,000$.

$$3000(4^T)+1000(16^T)=10000$$
$$3000(4^T)+1000(4^{2T})=10000$$
$$3(4^T)+4^{2T}=10 \quad \text{M1}$$
Let $y=4^T$
$$3y+y^2=10$$
$$y^2+3y-10=0$$
$$(y-2)(y+5)=0 \quad \text{A1}$$
A1 $y=2$ or $y=-5$ (rejected)
$$4^T=2$$
$$T=\frac{1}{2} \quad \text{A1}$$

10(i)

$$(1+2x)^4=(1)^4+\binom{4}{1}(1)^3(2x)^1+\binom{4}{2}(1)^2(2x)^2+\binom{4}{3}(1)^1(2x)^3+\binom{4}{4}(1)^0(2x)^4 \quad \text{M2}$$
$$=1+8x+24x^2+32x^3+16x^4 \quad \text{A1}$$

10.(ii)

x^9 term is $\binom{10}{9}(2)^1(x)^9=kx^9$, $\Rightarrow k=20$
M2 A1

10.(iii)

x^{13} term from $32x^3(x^{10})$ and $16x^4(kx^9)$ M1

Coefficient of x^{13} is $32+16k$

$= 32+16(20) = 352$ A1

11.

(i)

$$\frac{\sin 45^\circ}{\cos 30^\circ + \sin 60^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}$$ (M1)

$$= \frac{1}{\sqrt{6}}$$ (A1)

11.(ii)

$$\sin \frac{\pi}{3} \cos \frac{\pi}{6} + \cos \frac{\pi}{3}$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}$$ (M1)

$$= \frac{3}{4} + \frac{1}{2}$$

$$= \frac{5}{4}$$ (A1)

12.

$$AC^2 = 1 - x^2$$
$$AC = \sqrt{1 - x^2}$$ (B1)

(i) $\cos B = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$ (A1)

(ii) $\tan B = \frac{x}{\sqrt{1-x^2}}$ (A2)

13(i)

$$\text{Gradient of } AB = \frac{8-0}{0-(-10)}$$

$$= \frac{4}{5}$$ M1

Gradient of AB = Gradient of DC

Since CD passes through the origin, $c = 0$

$\therefore$ equation of CD is $y = \frac{4}{5}x$. A1

13.(ii)

$$\text{Gradient of } AD = -3 \times \frac{4}{5}$$

$$= -\frac{12}{5}$$ M1

the equation of AD is

$y = -\frac{12}{5}x + c$, where c is a constant.

Since $(-10, 0)$ is on the line,

$$0 = -\frac{12}{5}(-10) + c$$

$\therefore c = -24$ A1

$\therefore$ equation of AD is $y = -\frac{12}{5}x - 24$. A1

13.(iii)

Equation of BC *(as given)*: $y = -\dfrac{5}{4}x + 8$ -------------- (1)

Equation of CD: $y = \dfrac{4}{5}x$ -------------- (2)

Putting (2) into (1),

$$\frac{4}{5}x = -\frac{5}{4}x + 8$$

$$\frac{41}{20}x = 8$$ M1

$$x = \frac{160}{41}$$

$$y = \frac{4}{5}\left(\frac{160}{41}\right)$$

$$= \frac{128}{41}$$

$\therefore$ the coordinates of C is $\left(\dfrac{160}{41}, \dfrac{128}{41}\right)$ A1

13.(iv)

To find D.

Point D is where AD and CD meet.

Equation of AD: $y = -\dfrac{12}{5}x - 24$ -------------- (3)

Equation of CD: $y = \dfrac{4}{5}x$ -------------- (2)

M1

Putting (2) into (3),

$$\frac{4}{5}x=-\frac{12}{5}x-24$$

13.(iv) $\frac{16}{5}x=-24$

$$x=-\frac{15}{2}$$

$$y=\frac{4}{5}\left(-\frac{15}{2}\right)$$

$$=-6$$

$\therefore$ the coordinates of D is $\left(-7\frac{1}{2},-6\right)$. A1

$$\text{Area of trapezium } ABCD=\frac{1}{2}\begin{vmatrix}0 & -10 & -\frac{15}{2} & \frac{160}{41} & 0\\ 8 & 0 & -6 & \frac{128}{41} & 8\end{vmatrix}$$

$$=\frac{3510}{41}\,units^2$$

$$=85\frac{25}{41}\,units^2$$ A1

14.

<table>
<tr><td>(i)</td><td>

$\frac{x}{y-2}$	4	5	6	7.02	7.94
x	1	2	3	4	5

B1

$y=\frac{x}{ax+b}+2$
$y-2=\frac{x}{ax+b}$
$\frac{x}{y-2}=ax+b$

Gr
</td><td></td></tr>
<tr><td>(ii)</td><td>
From the graph,
$\frac{x}{y-2}$ intercept = 3
$b=3\pm0.1$ B1
The gradient = $a=1\pm0.1$ B1
</td><td></td></tr>
<tr><td>(iii)</td><td>
$2(x+9)=9y$
$2x+18=9y$
$2x=9(y-2)$
$\frac{x}{y-2}=4.5$ M1
$x=1.$ A1
</td><td></td></tr>
</table>

14.