Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2015
Secondary Two

INTEGRATED MATHEMATICS 1 hour 30 minutes

Paper 1

Thursday **8 October 2015** **0800 - 0930**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Write your name, register number and class in the spaces at the top of this page.
2. Answer **all** the questions.
3. Write your answers and working in the spaces provided on the question paper.
4. **All working must be written in dark blue or black ink.**
5. **Omission of essential working will result in loss of marks.**
6. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
7. The use of calculators is allowed for this paper.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 60.
3. You are reminded of the need for clear presentation in your answers.

1. Arrange the following numbers in ascending order.

$$3.2\times10^{-16}, -32\times10^{-15}, 0.32\times10^{-17}, -3.2\times10^{-13}.$$

Answer: _____ [2]

2. Solve the following pair of simultaneous equations:

$$3x - y = 10$$

$$\frac{x}{2} + 2y = 6$$

Answer: $x =$ _____ ; $y =$ _____ [3]

3. One cubic box of length 40 cm contains 4000 SG50 commemorative coins. A truck container with dimensions 2.5 m by 6 m by 2.5 m is used to deliver the coins to the packing center. Find the maximum number of coins that can be delivered each time. Leave your answer in standard form.

Answer:_____coins [2]

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4. Given that $p = 2 \times 10^{10}$ and $q = 1.62 \times 10^{12}$, evaluate each of the following without the use of a calculator. Express your answers in standard form.

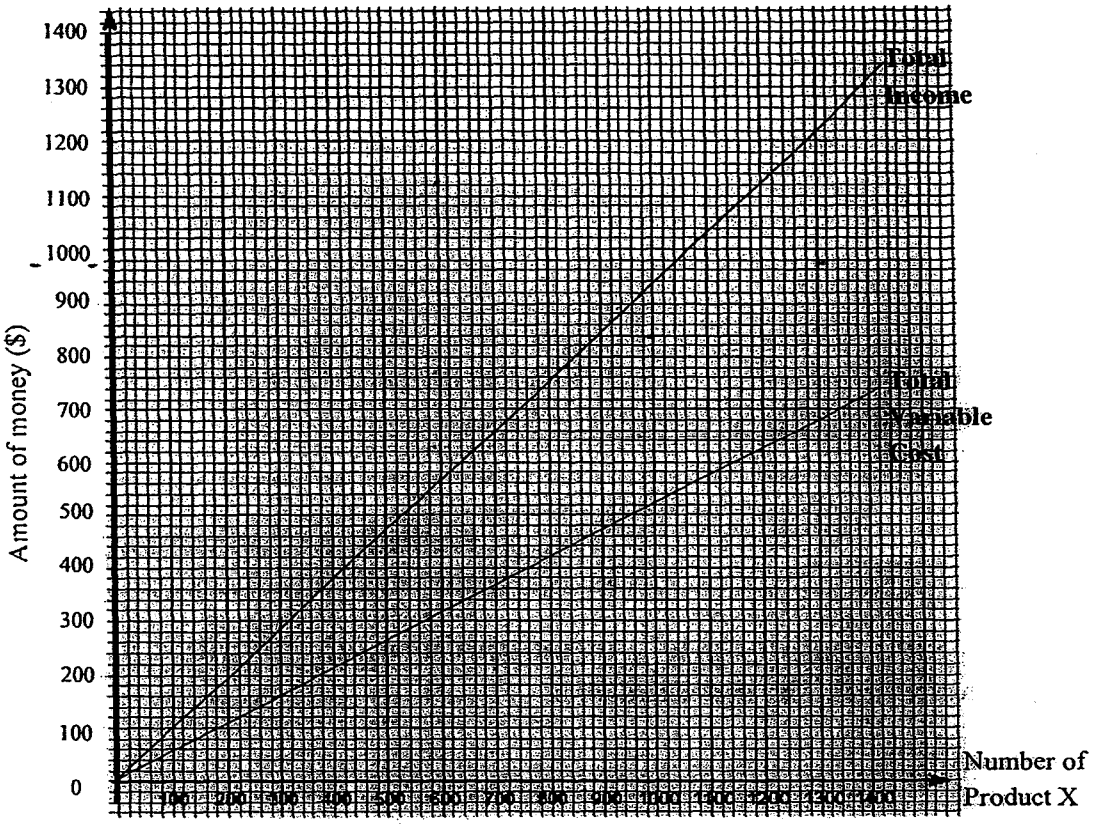
   (a) $8p$

   (b) $\dfrac{p^2}{q - p}$

Answer : (a)_____ [1]

(b)_____ [3]

5. The diagram below shows the Total Variable Cost and Total Income graphs of a company producing product X.



The Total Variable Cost includes expenses such as electricity bills, material costs, etc.

(a) Find the gradient of the line representing the Total Variable Cost and explain its significance.

The Total Cost incurred by the company comprises of the Total Variable Cost and the Fixed Cost. The Fixed Cost is $500 and includes expenses such as insurance fees, rental fees, etc.

(b) (i) On the graph provided above, draw the line representing the Total Cost for $0 \leq$ number of Product X ≤ 1400. Label your graph "Total Cost". [1]

(ii) State the Total Cost incurred by the company when 800 Product X are produced.

(iii) Hence, conclude whether the company is making a profit or loss when 800 Product X are produced.

Answer : (a) Gradient = _____;

Significance : _____ [2]

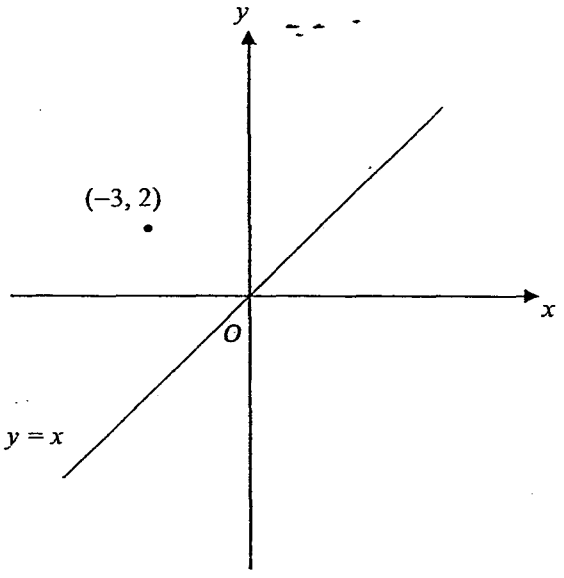
(b) (ii) $_____ [1] (iii)_____ [1]

6. The graph of $y = x$ and the point $(-3, 2)$ are plotted as shown below. On the axes below, sketch the graphs of the following given equations and, for each case, indicate the intercepts with the axes clearly. Label each graph clearly with its equation.

(a) $x = -4$, [1]

(b) $y + x + 1 = 0$, [2]

(c) $y = k + kx$, where $k > 1$. [2]



7. **(a)** Given that $b(3a - b) = \frac{ac}{b}$, express a in terms of b and c.

(b) State the range of values of x for the following equation to be defined.

$$\frac{1}{x\sqrt{x+1}} = \frac{1}{x+1}$$

Answer : (a)_____ [3]

(b)_____ [2]

[Turn Over

8. Simplify the following expressions leaving your answers in the simplest factorized form.

(a) $\dfrac{1}{2x-1}-\dfrac{3}{4x-2}+\dfrac{2}{4x^2-4x+1}$

(b) $\dfrac{3a^2-5ab-2b^2}{b^2-9a^2}\div\dfrac{ab-3a-2b^2+6b}{3}$

Answer : (a)_____[4]

(b)_____[4]

9. Simplify the following, giving your answers in positive indices only.

(a) $\left(\frac{a^3}{27b}\right)^{\frac{2}{3}} \times \frac{b}{(-a)^2}$

(b) $\frac{(a^{-2}b^3c^0)^2}{5a^3c^2} \times \sqrt{100b^{-8}}$

(c) $\frac{3^{2x+1} - 2(9^x)}{3^x}$, where $x > 1$

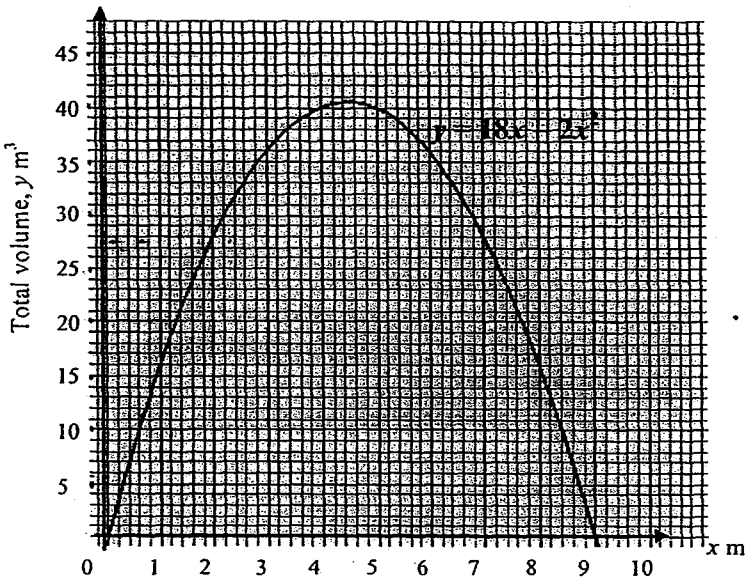
Answer : (a)_____ [3]

(b)_____ [3]

(c)_____ [2]

[Turn Over

10. A solid cuboid has dimensions $(9-x)$ m by x m by 2 m. The graph representing its total volume, $y=(18x-2x^2)\ \text{m}^3$, against x m is shown below.



(a) Find the value of m given that the points $(2, n)$ and (m, n) lie on the curve.

(b) State the significance of the point (4.5, 40.5) on the above graph.

(c) Find the equation of the straight line that must be drawn on the above graph to solve

$$x^2-\frac{21}{2}x+15=0.$$

(d) Another cuboid with x m as one of its sides has a total volume of $y=(30-3x)\ \text{m}^3$. By inserting the straight line $y=30-3x$ onto the graph above, find the value of x, where $x<5$, when the two cuboids have the same volume.

Answer : (a) $m=$ _____ [1]

(b) _____

_____ [1]

(c) _____ [2]

(d) _____ [2]

11. The following is a pair of simultaneous equations:

$$x^2 - py^2 = 0,$$
$$x - 2y = 0.$$

(a) If $x = p + 1$ and $y = q$, is the solution set of the above simultaneous equations, find the values of p and of q.

(b) Write down a linear equation such that it has

(i) an infinite number of solutions with $x - 2y = 0$,

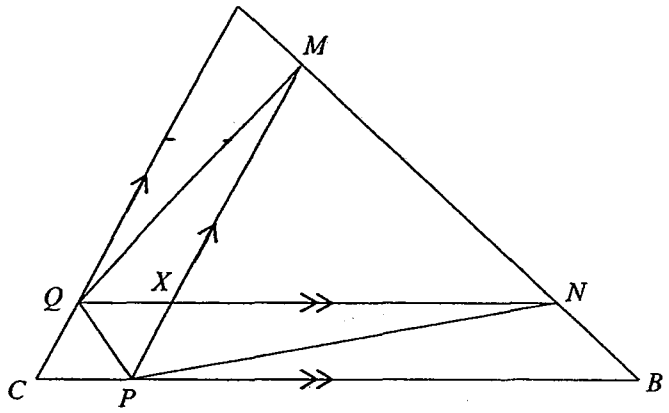
(ii) no solution with $x - 2y = 0$.

Answer : (a)_____ [5]

(b) (i) _____ (ii)_____ [2]

[Turn Over

12. In the diagram shown below, ABC is a triangle. The points M and P are on AB and BC respectively, such that PM is parallel to CA. The points N and Q are on AB and AC respectively, such that QN is parallel to CB. NQ and MP meet at the point X, such that $QXPC$ is a rhombus.



(a) Name a triangle which is similar to ΔMNX .

(b) What type of triangle should ΔNXM be, for it to be similar to ΔQXP ?

(c) Given $XN : QN : CB = 3 : 4 : 5$.

(i) Prove that ΔANQ and ΔMBP are congruent. State the geometrical reasons and the case of congruence clearly. [2]

(ii) If ΔQXP and ΔNXM are indeed similar, state the ratio $QP : MN$.

Answer:

(a)_____ [1]

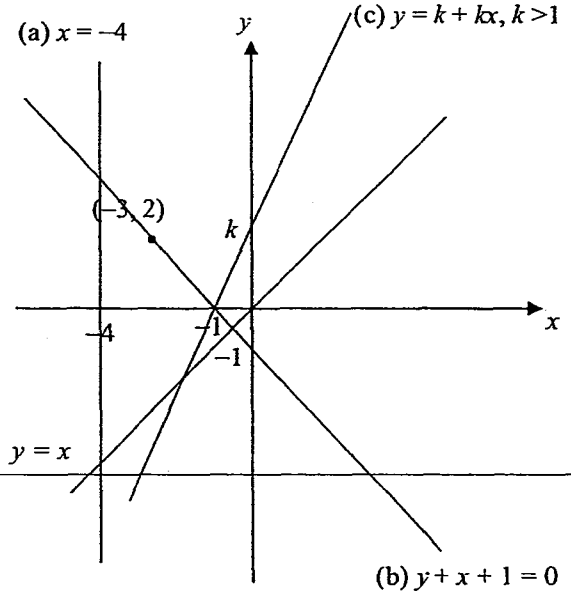
(b)_____ [1]

(c)(ii)_____ [1]

END OF PAPER

Sec 2 EOY Paper 1 Solution

1	$-3.2\times10^{-13}, -32\times10^{-15}, 0.32\times10^{-17}, 3.2\times10^{-16}$
2	$3x-y=10$ ---------(1) $\frac{x}{2}+2y=6$ ---------(2) $2(1)+(2)$: $2(3x)+\frac{x}{2}=2(10)+6$ $\frac{13x}{2}=26$ $x=4$ Sub $x=4$ into (1), $3(4)-y=10$ $y=2$ <u>Alternative method</u> $3x-y=10$ ---------(1) $\frac{x}{2}+2y=6$ ---------(2) From (1) : $y=3x-10$ -------(3) Sub (3) into (2) : $\frac{x}{2}+2(3x-10)=6$ $\frac{13x}{2}=26$ $x=4$ Sub $x=4$ into (3), $3(4)-y=10$ $y=2$
3	Maximum of boxes that can be packed into the truck container $=\frac{2.5}{0.4}\times\frac{6}{0.4}\times\frac{2.5}{0.4}$ $\approx 6\times15\times6$ $=540$ Maximum number of coins that can be delivered each time

	$= 540 \times 4000$ $= 2160000$ $= 2.16 \times 10^6$
4(a)	$8p$ $= 8(2 \times 10^{10})$ $= 16 \times 10^{10}$ $= 1.6 \times 10^{11}$
4(b)	$\frac{p^2}{q-p}$ $= \frac{(2 \times 10^{10})^2}{1.62 \times 10^{12} - 2 \times 10^{10}}$ $= \frac{4 \times 10^{20}}{1.62 \times 10^{12} - 0.02 \times 10^{12}}$ $= \frac{4 \times 10^{20}}{1.6 \times 10^{12}}$ or $\frac{4 \times 10^{20}}{160 \times 10^{10}}$ $= 2.5 \times 10^8$
5(a)	Gradient $= \frac{420}{800} = 0.525$ Significance: For each product X being produced, \$0.53 (2d.p) of variable cost was made.
5(bi)	Line drawn is <u>parallel</u> to Total Variable Cost line with <u>y-int=500</u>
5(bii)	420+500=\$920
5(biii)	loss
6	

7(a)	$b(3a-b)=\frac{ac}{b}$ $3a-b=\frac{ac}{b^2}$ $3a-\frac{ac}{b^2}=b$ $a\left(\frac{3b^2-c}{b^2}\right)=b$ $a=\frac{b^3}{3b^2-c}$ Alternative method: $b(3a-b)=\frac{ac}{b}$ $3ab^2-b^3=ac$ $3ab^2-ac=b^3$ $a(3b^2-c)=b^3$ $a=\frac{b^3}{3b^2-c}$
7(b)	$x\neq 0$ and $x>-1$ (Alt) $-1<x<0$ or $x>0$
8(a)	$\frac{1}{2x-1}-\frac{3}{4x-2}+\frac{2}{4x^2-4x+1}$ $=\frac{1}{2x-1}-\frac{3}{2(2x-1)}+\frac{2}{(2x-1)^2}$ $=\frac{2(2x-1)-3(2x-1)+2(2)}{2(2x-1)^2}$ $=\frac{5-2x}{2(2x-1)^2}$
8(b)	$\frac{3a^2-5ab-2b^2}{b^2-9a^2}\div\frac{ab-3a-2b^2+6b}{3}$ $=\frac{(a-2b)(3a+b)}{(b-3a)(b+3a)}\times\frac{3}{(a-2b)(b-3)}$ $=\frac{3}{(b-3a)(b-3)}$

9(a)	$\left(\frac{a^3}{27b}\right)^{\frac{2}{3}} \times \frac{b}{(-a)^2}$ $= \frac{a^2}{9b^{\frac{2}{3}}} \times \frac{b}{a^2}$ $= \frac{1}{9} b^{\frac{1}{3}}$
9(b)	$\frac{(a^{-2}b^3c^0)^2}{5a^3c^2} \times \sqrt{100b^{-8}}$ $= \frac{a^{-4}b^6}{5a^3c^2} \times 10b^{-4}$ $= \frac{2b^2}{a^7c^2}$
9(c)	$\frac{3^{2x+1} - 2(9^x)}{3^x}$ $= \frac{3(3^{2x}) - 2(3^{2x})}{3^x}$ $= \frac{3^{2x}}{3^x}$ $= 3^x$
10(a)	$m=7$
10(b)	The maximum total volume is 40.5 m^3 when $x=4.5$.
10(c)	$x^2 - \frac{21}{2}x + 15 = 0$ $2x^2 - 21x + 30 = 0$ $30 - 3x = 18x - 2x^2$ $y = 30 - 3x$
10(d)	Line drawn passes through (0,30) and (10, 0). Accept $x = 1.6$ to 1.8

11(a)	$(p+1)^2 - pq^2 = 0$ ----------(1) $(p+1) - 2q = 0$ ----------(2) From (2), $q = \frac{p+1}{2}$ ----------(3) Sub (3) into (1), $(p+1)^2 - \frac{p(p+1)^2}{4} = 0$ $(p+1)^2\left[1 - \frac{p}{4}\right] = 0$ $p = -1$ or $p = 4$ From (3), If $p = -1, q = 0$ If $p = 4, q = 2.5$ <u>Alternative method:</u> $(p+1)^2 - pq^2 = 0$ ----------(1) $(p+1) - 2q = 0$ ----------(2) From (2), $p = 2q - 1$ ----------(3) Sub (3) into (1), $(2q)^2 - (2q-1)q^2 = 0$ $q^2(5-2q) = 0$ $q = 0$ *or* $q = 2.5$ From (3), If $q = 0, p = -1$ If $q = 2.5, p = 4$
11(b)	(i) accept any $nx - 2ny = 0$, where $n \neq 0$ (ii) accept any $x - 2y = n$, where $n \neq 0$
12(a)	ΔANQ or ΔMBP or ΔABC
12(b)	Isosceles / Equilateral triangle

12(ci)	$\angle QAN = \angle PMB$ (corr. $\angle$s, $AC \parallel MP$) $\angle ANQ = \angle MBP$ (corr. $\angle$s, $QN//PB$) $\angle AQN = \angle MXN$ (corr. $\angle$s, $AQ//MP$) $= \angle MPB$ (corr. $\angle$s, $AN//PB$) (*any of the 2 pairs of angles) $QX = CP$ $QN : PB = 4 : 5 - (4 - 3)$ $= 4 : 4$ $QN = PB$ $\Delta ANQ \equiv \Delta MBP$ (AAS/ASA * depends)
12(cii)	$QP : MN = 1 : 3$

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南洋女子中学校
NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2015
Secondary Two

INTEGRATED MATHEMATICS 1 hour 30 minutes
Paper 2
1030 - 1200 **08 October 2015**

READ THESE INSTRUCTIONS FIRST

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7. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
8. The use of calculators is allowed for this paper.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 60.
3. You are reminded of the need for clear presentation in your answers.

1 **(a)** The point $(3, k)$ lies on the line $y = 3x + 1$. Find the value of k. [1]

(b) The line L_1 passes through the point (4, 7) and is parallel to the line $2y - x = 16$. Find the equation of the line L_1. [3]

(c) The line L_2 passes through the points (2, –2) and (2, 7). Write down the equation of the line L_2. [1]

2 **(a)** It is given that $\frac{3x-1}{4} \leq \frac{7x+4}{3} < x + 2\frac{2}{3}$.

(i) Solve the inequality. [4]

(ii) Hence, list the integer values of x that satisfy the inequality. [1]

(b) Given that $2 \leq p \leq 7$ and $-1 \leq q \leq 5$, find

(i) the largest value of $p - q$, [1]

(ii) the smallest value of $p + q^2$, [1]

(iii) the smallest value of $\frac{q^3}{p}$. [1]

3 A delivery van runs x kilometres on each litre of petrol when it travels up a slope.

(i) Write down, in terms of x, the number of litres of petrol used when the delivery van travels 70 km up the slope. [1]

The delivery van runs $(x + 2)$ kilometres on each litre of petrol when it travels down the slope.

(ii) Write down, in terms of x, the number of litres of petrol used when the delivery van travels 70 km down the slope. [1]

The delivery van uses 3 litres less petrol to travel down the slope than up the slope.

(iii) Using this information, form an equation in x and show that it reduces to $3x^2 + 6x - 140 = 0$. [3]

(iv) Solve the equation $3x^2 + 6x - 140 = 0$, giving both answers correct to two decimal places. [3]

(v) Hence, calculate the total volume of petrol used when the van travels 70 km up the slope and 70 km down the slope. [2]

4 The quadratic curve $y = ax^2 + bx + 23$ cuts the y-axis at point A and it passes through the points (1, 13) and (5, 13).

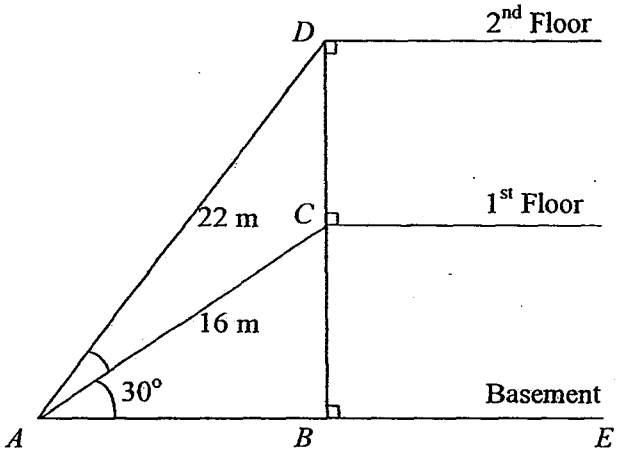
(i) Write down the coordinates of point A. [1]

(ii) Find the equation of the line of symmetry of the curve. [2]

(iii) Find the value of a and of b. Hence, explain with a reason whether the curve has a maximum or minimum turning point. [5]

(iv) The line $y = k$ meets the curve $y = ax^2 + bx + 23$ at only one point. Find the value of k. [2]

5 The diagram shows two escalators, AC and AD, in a shopping centre.



The escalator AC of length 16 m leads to the 1st Floor while the escalator AD of length 22 m leads to the 2nd Floor.

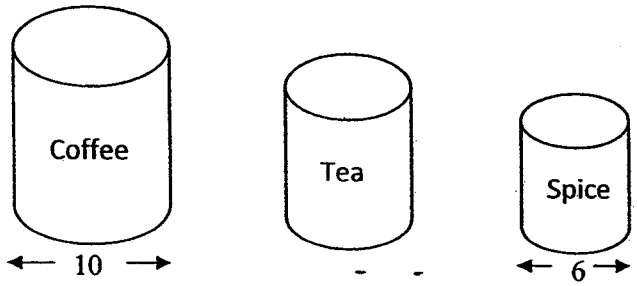
Given that the straight line BCD is perpendicular to the horizontal basement ABE and $\angle BAC = 30°$, find

(i) the height between the basement and the 1st floor, [2]

(ii) the height between the 1st floor and the 2nd floor, [4]

(iii) $\angle DAC$. [2]

6 The diagram shows three kitchen containers.



Each container is a cylinder and the containers are geometrically similar.

The bases of the Coffee and Spice containers have diameters of lengths 10 cm and 6 cm respectively.

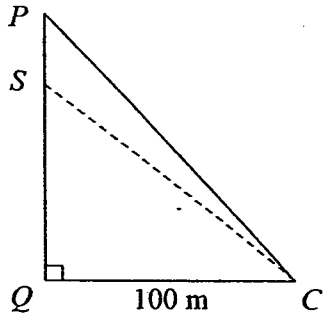
(a) Calculate the ratio

volume of the Spice container : volume of the Coffee container.

Hence, find the volume of the Spice container if the volume of the Coffee container is 980 cm^3. [3]

(b) Given that the surface area of the Tea container is $\frac{9}{16}$ of the surface area of the Coffee container, evaluate $\frac{\text{Surface area of the Spice container}}{\text{Surface area of the Tea container}}$. [3]

7 At the National Day Parade, a parachutist descended from a helicopter (not shown in the diagram). When he was at point P, he began to descend vertically at a constant speed of 7 m/s towards point Q on the parade ground. To record the descent, an automated motorized video camera was placed at point C on the parade ground, 100 m away from point Q. The angle of depression of C from P was 60°.



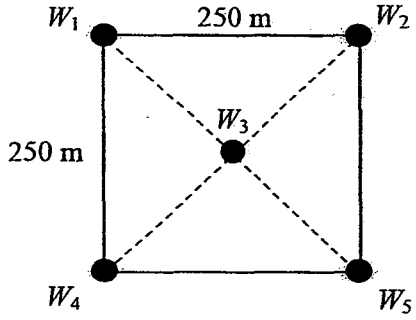
(i) Calculate the time taken for the parachutist to reach the parade ground. [3]

(ii) Given that in the descent, it took 10 s for the parachutist to reach point S, calculate the angle of elevation of S from C. [3]

8 Singapore aims to be a test-bed for micro-wind technology generating electricity with low wind speeds. A wind power station is proposed to be erected in a 250 m by 250 m horizontal field at Pulau Ubin and some wind towers will be built. Each wind tower has three rotor blades and the length of each blade is 40 m.

(a) According to building regulations, the minimum distance between two wind towers, measured from the foot of one tower to another, has to be five times the length of a rotor blade.

An engineer made a suggestion on how to arrange five wind towers W_1, W_2, W_3, W_4 and W_5, in the square field. The arrangement is shown in the diagram below.



Explain why the engineer's suggestion does not meet the building regulations. Support your answer with working. [3]

(b) Singapore wants to estimate the cost savings from generating electricity through this wind station. The formula $C = -10y^2 + 90y - 130$ is used to estimate the cost savings in C million dollars during the first y years of operation.

This formula can be re-written as $C = a(y-n)^2 + m$ where it will take n years to achieve the maximum cost savings of m million dollars. Find the maximum cost savings and the number of years it will take to achieve this. [4]

Bonus Question

9 α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$ where a, b and c are constants and $a \neq 0$. Showing your working clearly, express the sum of the roots and the product of the roots in terms of a, b and/or c. [3]

End of Paper 2

Sec 2 Math EOY 2015 Paper 2 Solution

1(a) [1 m]	$k = 10$
1(b) [3 m]	Gradient of L_1 is $\frac{1}{2}$ Let the equation of the line be $y = mx + c$ and since the line passes through (4, 7) $7 = \frac{1}{2}(4) + c$ $\therefore c = 5$ Equation of L_1 is $y = \frac{1}{2}x + 5$
1(c) [1 m]	$x = 2$
2(a)(i) [4 m]	$\frac{3x-1}{4} \le \frac{7x+4}{3} < x + 2\frac{2}{3}$ $\frac{3x-1}{4} \le \frac{7x+4}{3}$ $9x - 3 \le 28x + 16$ $-1 \le x$ or $x \ge -1$ $\frac{7x+4}{3} < x + 2\frac{2}{3}$ $7x + 4 < 3x + 8$ $x < 1$ $\therefore -1 \le x < 1$
2(a)(ii) [1 m]	-1 and 0
2(b)(i) [1 m]	8
2(b)(ii) [1 m]	2
2(b)(iii) [1 m]	$-\frac{1}{2}$
3(i) [1 m]	$\frac{70}{x}$
3(ii) [1 m]	$\frac{70}{x+2}$

3(iii) [3 m]	$\frac{70}{x}-\frac{70}{x+2}=3$ $70(x+2)-70x=3x(x+2)$ $70x+140-70x=3x^2+6x$ $3x^2+6x-140=0$ (shown)
3(iv) [3 m]	$x=\frac{-6\pm\sqrt{6^2-4(3)(-140)}}{2(3)}$ $x\approx 5.90$ or -7.90
3(v) [2 m]	$\frac{70}{5.904}+\frac{70}{5.904+2}$ ≈ 20.7 litres
4(i) [1 m]	(0, 23)
4(ii) [2 m]	$x=\frac{1+5}{2}$ Line of symmetry is $x=3$
4(iii) [5 m]	$a+b+23=13$ $a+b=-10$ .....(1) $25a+5b=-10$ .....(2) $25a+5(-10-a)=-10$ or $25a+5b-5a-5b=-10-5(-10)$ $20a=-10+50$ $a=2$ From (1), b = $-10-2=-12$ Since a, the *coefficient of x^2, is positive*, the curve has a minimum turning point.
4(iv) [2 m]	When $x=3$ $k=2(3)^2-12(3)+23$ $\therefore k=5$
5(i) [2 m]	$BC=16\sin 30°$ $=8$ m
5(ii) [4 m]	$AB=16\cos 30°$ ≈ 13.86 m $BD=\sqrt{22^2-13.86^2}$ ≈ 17.09 m $CD=17.09-8$ $=9.09$ m

5(iii) [2 m]	$\angle BAD = \cos^{-1}\frac{13.86}{22}$ $\approx 50.95°$ $\angle DAC = 50.95° - 30° \approx 21.0°$ or $20.9°$
6(a) [3 m]	$6^3 : 10^3$ $= 27 : 125$ Volume of Coffee container $= \frac{27}{125} \times 980$ $= 211.68\,\text{cm}^3$
6(b) [4 m]	$\frac{\text{Surface area of Spice container}}{\text{Surface area of Coffee container}} = \left(\frac{3}{5}\right)^2$ $= \frac{9}{25}$ $\frac{\text{Surface area of Spice container}}{\text{Surface area of Tea container}} = \frac{9}{25} \div \frac{9}{16}$ $= \frac{16}{25}$
7(i) [3 m]	$\frac{QP}{100} = \tan 60°$ $QP = 173.21$ Time taken $= \frac{173.21}{7}$ $\approx 24.7\,\text{s}$
7(ii) [3 m]	$QS = 173.21 - 10(7)$ $= 103.21$ Angle of Elevation of S from C $= \tan^{-1}\frac{103.21}{100}$ $\approx 45.9°$
8(a) [3 m]	$\frac{\sqrt{250^2 + 250^2}}{2}$ ≈ 177 Since $177 < 200$, the engineer's suggestion does not meet the building requirements.
8(b) [4 m]	$-10(y^2 - 9y) - 130$ $= -10(y^2 - 9y + 4.5^2 - 4.5^2) - 130$ $= -10(y - 4.5)^2 + 72.5$ It takes 4.5 years to achieve a maximum cost

	savings of 72.5 million dollars
9	$(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$ $x^2 - (\alpha+\beta)x + \alpha\beta \equiv x^2 + \frac{b}{a}x + \frac{c}{a}$ $\therefore \alpha+\beta = -\frac{b}{a}$ $\alpha\beta = \frac{c}{a}$